Modeling Toll Roads with EMME

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Motivation

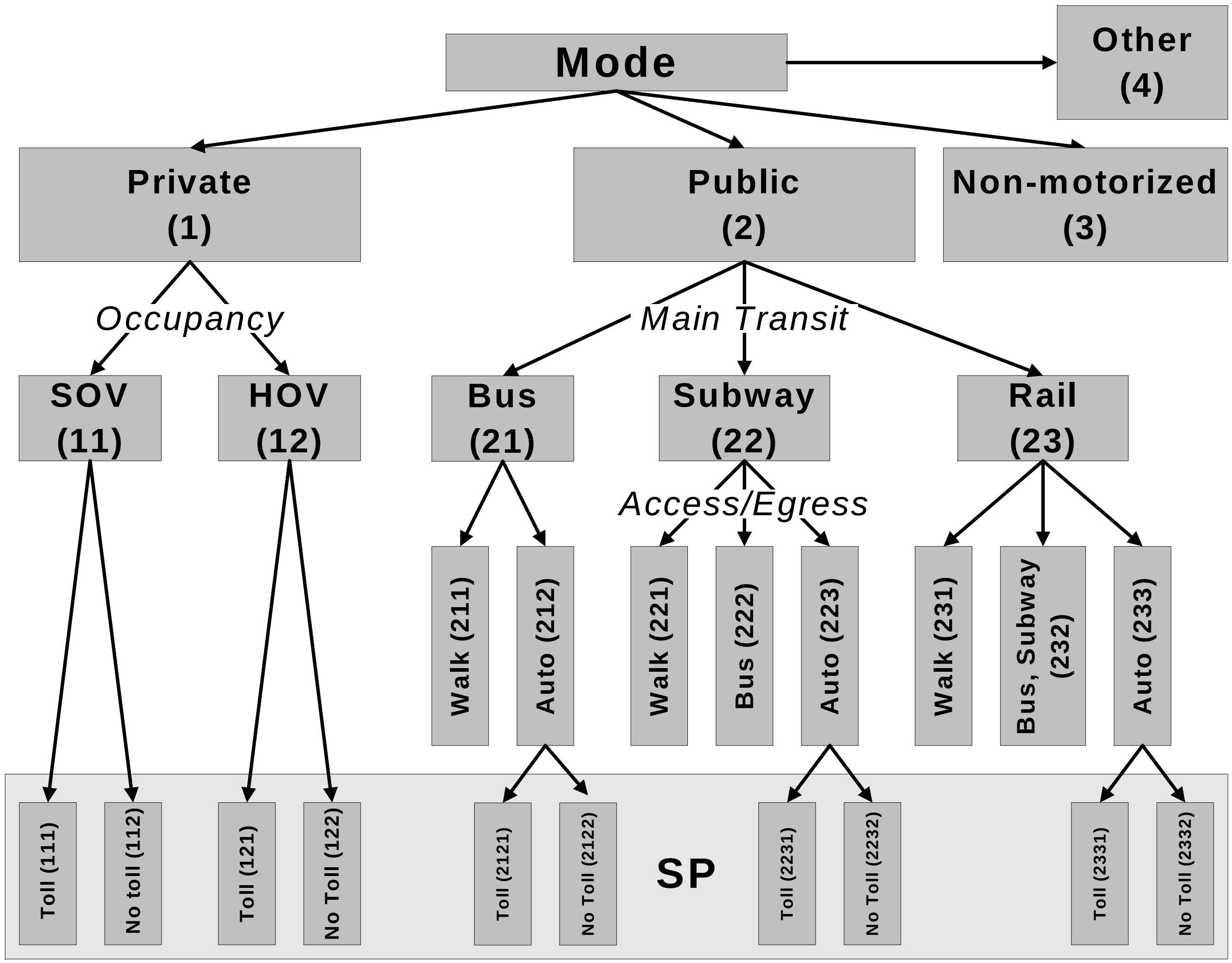
- Lessons learned from MTQ Traffic & Revenue Study for Montreal Region:
  - Theoretical issues
  - Practical limitations and compromises
  - Relation of practical “tricks” to theory
- Road pricing is the “hottest” topic:
  - Broad framework of modeling issues
  - Operational implementation with EMME

1. Model System Framework

Existing Approaches

- Travel demand dimensions:
  - Assignment-only with fixed demand and generalized cost (route choice w/network attributes)
    - Can be combined with other choices (mode, TOD, destination)
  - Binary pre-route choice (toll/non-toll) integrated with multi-class assignment (route choice w/additional attributes)
    - Can be combined with other choices (mode, TOD, destination)
  - Variable demand (diversion curve) generalized-cost assignment (route choice w/network attributes and crude representation of all other choices)
- Network simulation tool:
  - Static deterministic UE for large regional networks
  - DTA/microsimulation for corridor studies

Travel Choice Hierarchy



Traffic Simulation

Initial matrices	Auto		Comm		Light		Heavy	
Initial split	A/n	A/t	C/n	C/t	L/n	L/t	H/n	H/t

Multi(8)-class assignment

Freeze link travel times

Skimming	A/n	A/t	C/n	C/t	L/n	L/t	H/n	H/t
Binary choice	A/n	A/t	C/n	C/t	L/n	L/t	H/n	H/t

Travel model

Traffic Simulation

- 8-class assignment with 1,600 zones for 4 TOD periods at each global iteration 1,2,3…
- Travel demand model (mode and binary pre-route choice):
  - Auto matrices:
    - toll users
    - non-toll users
- Binary pre-route choice model w/fixed total demand:
  - Commercial vehicle matrices:
    - toll users
    - non-toll users
  - Light truck matrices:
    - toll users
    - non-toll users
  - Heavy truck matrices:
    - toll users
    - non-toll users

2. Core Formulation

Focus

- Convex program with objective function combining
  - Beckman-type link terms and
  - Entropy-type OD demand terms
- Demand choice hierarchy (mode, TOD, destination):
  - Adds complexity to entropy-type terms
  - Not essential for most issues related to tolls
- Simplified binary choice representation:
  - Useful for illustration of problems
  - Discrepancy between theory and practice

Formulation

Toll / Toll user matrix / Toll bias

$$\min\left\{\sum_{a\in A}\int_0^{v_a} c_a(v)\,dv+\sum_{a\in A}\sum_{k\in K}\frac{1}{\theta_k}t_a^k v_a^k+\sum_{b\in B}\int_0^{v_b} c_b(v)\,dv+\sum_{k\in K}\frac{1}{\beta^k}\left[X_{ij}^k\left(\ln X_{ij}^k-\alpha^k\right)+Y_{ij}^k\ln Y_{ij}^k\right]\right\}$$

Toll links / Classes / VOT / Non-toll links / Choice coefficient / Non-toll user matrix

Subject to:	**Toll**	**Non-toll**
Total demand	$X_{ij}^k+Y_{ij}^k=D_{ij}^k$	
Flow conservation	$\sum_{r\in R_{ij}^k} x_{ijr}^k = X_{ij}^k$	$\sum_{s\in S_{ij}^k} y_{ijs}^k = Y_{ij}^k$
Link volume as function of path flows	$v_a^k=\sum_{ij\in I\times J}\sum_{r\in R_{ij}^k}\delta_{ijr}^a x_{ijr}^k$	$v_b^k=\sum_{ij\in I\times J}\left(\sum_{r\in R_{ij}^k}\delta_{ijr}^b x_{ijr}^k+\sum_{s\in S_{ij}^k}\delta_{ijs}^b x_{ijs}^k\right)$
Total link volume	$v_a=\sum_{k\in K} v_a^k$	$v_b=\sum_{k\in K} v_b^k$
Flow non-negativity	$x_{ijr}^k\ge 0$	$y_{ijr}^k\ge 0$

Solution

Binary logit choice:	
$X_{ij}^{k} = D_{ij}^{k} \frac{\exp\left(\alpha^{k} - \beta^{k} u_{ij}^{k}\right)}{\exp\left(\alpha^{k} - \beta^{k} u_{ij}^{k}\right) + \exp\left(-\beta^{k} w_{ij}^{k}\right)}$	$Y_{ij}^{k} = D_{ij}^{k} \frac{\exp\left(-\beta^{k} w_{ij}^{k}\right)}{\exp\left(\alpha^{k} - \beta^{k} u_{ij}^{k}\right) + \exp\left(-\beta^{k} w_{ij}^{k}\right)}$

UE network conditions:		
$u_{ij}^{k} = g_{ijr}^{k} = c_{ijr}^{k} + t_{ijr}^{k} = \sum_{a \in A} \delta_{ijr}^{a} c_{a}\left(v_{a}\right) + \sum_{b \in B} \delta_{ijr}^{b} c_{b}\left(v_{b}\right) + \frac{1}{\theta^{k}} \sum_{a \in A} \delta_{ijr}^{a} t_{a}^{k}$	if	$x_{ijr}^{k} > 0$
$u_{ij}^{k} \le g_{ijr}^{k}$	if	$x_{ijr}^{k} = 0$
$w_{ij}^{k} = g_{ijs}^{k} = c_{ijs}^{k} = \sum_{b \in B} \delta_{ijs}^{b} c_{b}\left(v_{b}\right)$	if	$y_{ijs}^{k} > 0$
$w_{ij}^{k} \le g_{ijs}^{k}$	if	$y_{ijs}^{k} = 0$

Implementation Algorithm

Step	Toll	Non-toll
Step 0: Initialization $n = 0$	$X_{ij}^{k}(0)$	$Y_{ij}^{k}(0)$
Step 1: Assignment	$\min\left\{\sum_{a\in A}\int_{0}^{v_a} c_a(v)dv + \sum_{a\in A}\sum_{k\in K}\frac{1}{\theta_k}t_a^k v_a^k + \sum_{b\in B}\int_{0}^{v_b} c_b(v)dv\right\}$	
	$\sum_{r\in R_{ij}^{k}} x_{ijr}^{k} = X_{ij}^{k}(n)$	$\sum_{s\in S_{ij}^{k}} y_{ijs}^{k} = Y_{ij}^{k}(n)$
	$v_a^k = \sum_{ij\in I\times J}\sum_{r\in R_{ij}^{k}} \delta_{ijr}^{a} x_{ijr}^{k}$	$v_b^k = \sum_{ij\in I\times J}\left(\sum_{r\in R_{ij}^{k}} \delta_{ijr}^{b} x_{ijr}^{k} + \sum_{s\in S_{ij}^{k}} \delta_{ijs}^{b} x_{ijs}^{k}\right)$
	$v_a = \sum_{k\in K} v_a^k$	$v_b = \sum_{k\in K} v_b^k$
	$x_{ijr}^{k} \geq 0$	$y_{ijr}^{k} \geq 0$

Implementation Algorithm

Step	Toll	Non-toll
Step 2: Skimming	Time $c_{ij}^{k} = \frac{\sum_{r \in R_{ij}^{k}} c_{ijr}^{k} x_{ijr}^{k}}{X_{ij}^{k}(n)}$	Time $\tilde{c}_{ij}^{k} = \frac{\sum_{s \in S_{ij}^{k}} c_{ijs}^{k} y_{ijs}^{k}}{Y_{ij}^{k}(n)}$
	Cost $t_{ij}^{k} = \frac{\sum_{r \in R_{ij}^{k}} t_{ijr}^{k} x_{ijr}^{k}}{X_{ij}^{k}(n)}$	
	Generalized cost $u_{ij}^{k} = \frac{\sum_{r \in R_{ij}^{k}} g_{ijr}^{k} x_{ijr}^{k}}{X_{ij}^{k}(n)}$	Generalized cost $w_{ij}^{k} = \frac{\sum_{s \in S_{ij}^{k}} g_{ijs}^{k} y_{ijs}^{k}}{Y_{ij}^{k}(n)}$

Implementation Algorithm

Step	**Toll**	**Non-toll**
Step 3: Demand direction $n+=1$	$X_{ij}^{k}(n)=D_{ij}^{k}\dfrac{\exp\left(\alpha^{k}-\beta^{k}c_{ij}^{k}-\gamma^{k}t_{ij}^{k}\right)}{\exp\left(\alpha^{k}-\beta^{k}c_{ij}^{k}-\gamma^{k}t_{ij}^{k}\right)+\exp\left(-\beta^{k}\tilde{c}_{ij}^{k}\right)}$	
		$Y_{ij}^{k}(n)=D_{ij}^{k}\dfrac{\exp\left(-\beta^{k}\tilde{c}_{ij}^{k}\right)}{\exp\left(\alpha^{k}-\beta^{k}c_{ij}^{k}-\gamma^{k}t_{ij}^{k}\right)+\exp\left(-\beta^{k}\tilde{c}_{ij}^{k}\right)}$
	or	
	$X_{ij}^{k}(n)=D_{ij}^{k}\dfrac{\exp\left(\alpha^{k}-\beta^{k}u_{ij}^{k}\right)}{\exp\left(\alpha^{k}-\beta^{k}u_{ij}^{k}\right)+\exp\left(-\beta^{k}w_{ij}^{k}\right)}$	
		$Y_{ij}^{k}(n)=D_{ij}^{k}\dfrac{\exp\left(-\beta^{k}w_{ij}^{k}\right)}{\exp\left(\alpha^{k}-\beta^{k}u_{ij}^{k}\right)+\exp\left(-\beta^{k}w_{ij}^{k}\right)}$
	Two formulations are equivalent if $\gamma^{k}=\dfrac{\beta^{k}}{\theta^{k}}$	

Implementation Algorithm

Step	Toll	Non-toll
Step 4: Modify demand	$X_{ij}^{k}(n)=[1-\lambda(n)]X_{ij}^{k}(n-1)+\lambda(n)X_{ij}^{k}(n)$	$Y_{ij}^{k}(n)=D_{ij}^{k}-X_{ij}^{k}(n)$
Step 5: Convergence test and go to Step 1 if has not achieved		

3. Theoretical Issues

Uniqueness

- Unique:
  - Total link volumes v_a, v_b
  - Total cost in generalized terms $\sum_{a \in A} \sum_{k \in K} \frac{1}{\theta_k} t_a^k v_a^k$
- Not unique:
  - Path flows (and skims) x_{ijr}^k, y_{ijr}^k
  - Class-specific link volumes v_a^k, v_a^b
  - Total revenue in monetary terms $\sum_{a \in A} \sum_{k \in K} t_a^k v_a^k$

Sets of Routes

- Two fixed sets of routes are needed for each OD and class:
  - Toll routes $r \in R_{ij}^{k}$
  - Non-toll routes $s \in S_{ij}^{k}$
- Full sets of routes cannot be explicitly enumerated in real-world networks
- Constrained sets of routes cannot be formed in advance

Two Approaches

- Non-overlapping sets:
  - Choice is made at the trip level
  - Toll user matrix closely matches toll link volumes:
    - Non-toll users must use non-toll routes only
    - Toll users must use toll routes only
      - Restrictive assignment making toll route available for each OD
      - Constraining toll route set and considering unavailable toll route choice for some ODs where toll skim is 0
      - Promoting toll users to choose toll route
- Partially overlapping sets:
  - Choice is made at a strategic level (acquisition of transponder)
  - Toll user matrix can deviate from toll link volumes (“leaks”):
    - Non-toll users must use non-toll routes only
    - Toll users can use toll and non-toll routes
      - Promoting toll users to choose toll route

Which is Right?

- There is no theoretical contradiction in core formulation with either of approaches as far as
  - Sets are predetermined
  - Model was estimated, calibrated, and applied in consistent way
- However, practical implementation includes heuristics that makes results sensitive:
  - Toll route choice promotion – to what extent?
  - Interpretation of results and toll user “leaks”
  - Treatment of “fractional” users in multi-path skims

Toll Route Choice Promotion

- Find the shortest toll route for given OD pair in terms of generalized cost:
  - Zero out toll (or even travel time on toll link):
    - Helps avoid non-availability of toll route
    - No route (skim) distortion if toll value is known for each OD (single toll bridge) – users have already paid toll in binary choice!
    - Significant route (skim) distortion otherwise (parallel bridges with different tolls)
  - Apply real tolls:
    - Many OD pairs will not use toll facility (i.e. toll route is unavailable) if travel time on toll route is not significantly better than shortest non-toll route
    - Too restrictive in probabilistic choice framework (people choose longer and more expensive routes sometimes because of other factors like familiarity and safety)
    - No route (skim) distortion
  - Tempting to go into complications:
    - Multi-stage procedure with gradually reduced toll to grab more ODs
    - Behavioral thresholds in route choice

Binary Choice Utility

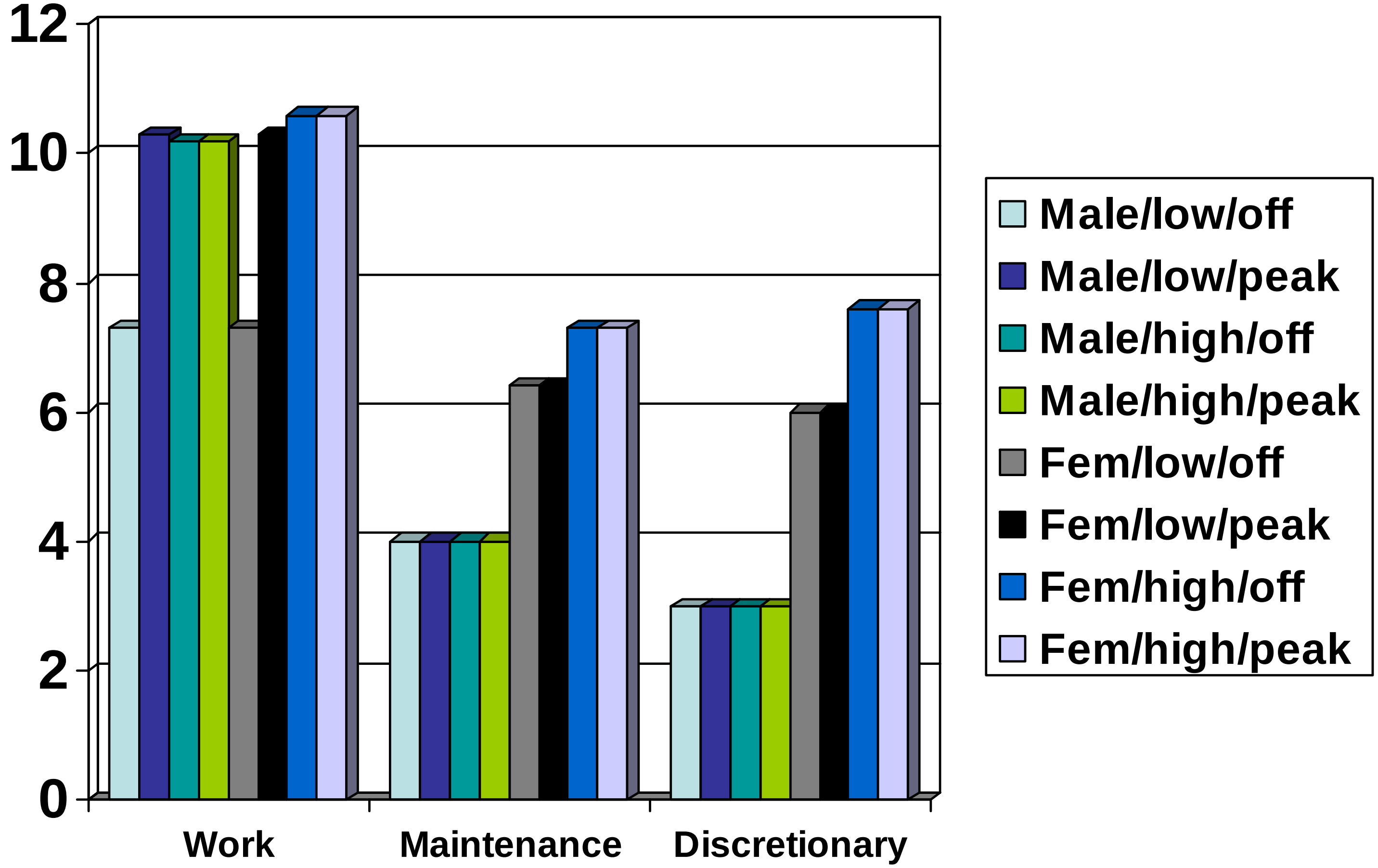
- Consistent with theoretical formulation:
  - Additive-by-link components of generalized cost:
    - Perceived time/distance VDF
      - Free flow time/distance
      - Congestion delay
    - Toll (fixed or volume-dependent)
  - Time and cost coefficients corresponding to VOT in generalized cost
  - Predetermined bias constant, O, D, or OD-attribute
- Inconsistent with theoretical formulation (but tempting!):
  - Non-linear transformations of route time and cost variables:
    - Logged travel time savings
    - Squared tolls
  - Non-additive-by-link skims:
    - Maximum V/C (proxy for reliability)
    - % of highway/freeway distance
  - Time and cost coefficients not corresponding to VOT
  - Entry-exit matrix toll

4. Practical Limitations & Compromises

Reasons for Multi-Class

Class	Prohibitions	Differential tolls	Differential VOT
SOV	X	X	X
HOV2	X	X	X
HOV3+		X	X
Commercial	X	X	X
Light truck	X	X	X
Heavy truck	X	X	X
Travel purpose			X
Income			X
Gender			X

VOT Estimates, $/h



Inevitable Compromise

- Core demand model has multiple travel and population segments
- Multiple segments are aggregated into feasible number of classes for assignment
- VOT is averaged within each class:
  - Aggregation bias in network simulation and
  - Discrepancy between demand model and network simulation

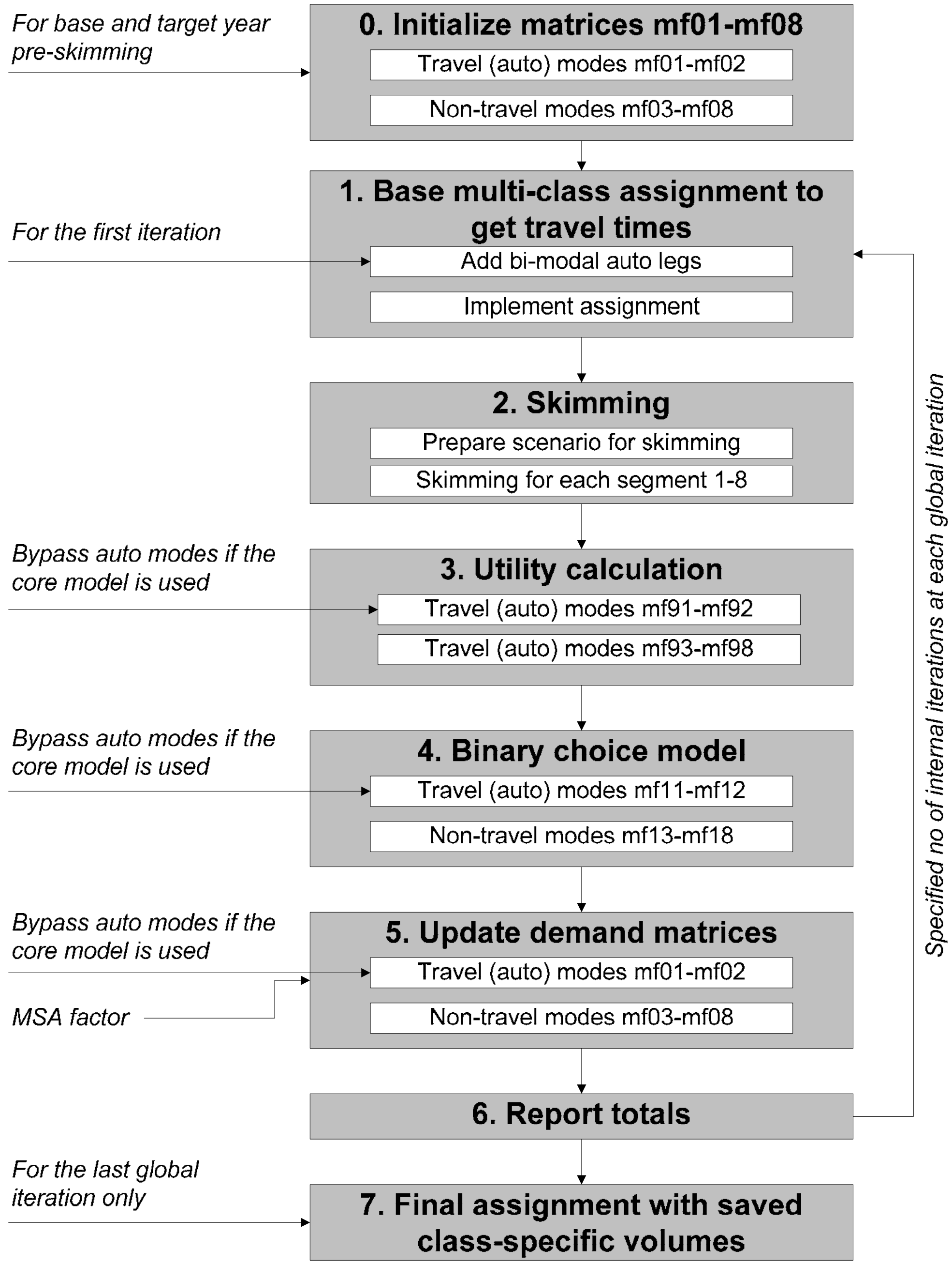
Directions for Improvement

- EMME software side:
  - Extend number of classes and take advantage of simultaneous processing of several classes with similar path building trees
  - Take advantage of matrix sparseness (a lot of zeros and we can make more of “discretizing”)
- Model implementation side:
  - Group segments by similar VOT and create as homogeneous classes as possible (“smart segmentation”)
  - Simulate probabilistic VOT as part of individual demand modeling and group the outcomes by VOT

Shortest Path Skimming

- Additional options multi-class assignment was limited to one class skim at a time
- Multiple repetition of multi-class assignment for skimming purpose was unrealistic
- Shortest path skimming was implemented based on link generalized cost frozen after assignment and used as flat VDF:
  - Good approximation for OD generalized cost
  - Bad and highly unstable approximation for time and cost/toll; devastating for convergence
  - Especially problematic if the binary pre-route choice model:
    - ✓ Use time and cost skims w/differential coefficients beyond network generalized cost framework
    - ✓ Toll route availability constraint is applied (non-overlapping sets)

Macro



For base and target year pre-skimming
0. Initialize matrices mf01-mf08
Travel (auto) modes mf01-mf02
Non-travel modes mf03-mf08
For the first iteration
1. Base multi-class assignment to get travel times
Add bi-modal auto legs
Implement assignment
2. Skimming
Prepare scenario for skimming
Skimming for each segment 1-8
Bypass auto modes if the core model is used
3. Utility calculation
Travel (auto) modes mf91-mf92
Travel (auto) modes mf93-mf98
Bypass auto modes if the core model is used
4. Binary choice model
Travel (auto) modes mf11-mf12
Non-travel modes mf13-mf18
Bypass auto modes if the core model is used
MSA factor
5. Update demand matrices
Travel (auto) modes mf01-mf02
Non-travel modes mf03-mf08
6. Report totals
Specified no of internal iterations at each global iteration
For the last global iteration only
7. Final assignment with saved class-specific volumes

Main Conclusion

With all limitations and compromises
the EMME-based model system
produced reasonable results,
stood numerous sensitivity tests, and
helped bring the project to the tender stage

More Conclusions

- What do modelers want?
  - Large regional networks w/high level of spatial resolution (4,000-5,000 zones and even more)
  - Numerous travel and population segments for better representation of behavior (purpose, income, gender, etc)
- What can realistically be done?
  - Take advantage of general hardware/software progress
  - Microsimulation concept:
    - Demand side: individual-based instead of matrix-based
    - Network side: probably the same?

New Challenges of Road Pricing

- Non-trip-based pricing forms:
  - Daily area pricing
  - Credit forms and bulk discounts
- Different toll-collection technologies:
  - Manual,
  - Pass/transponder
  - Automatic Vehicle Identification
- Perception of toll road quality beyond average time savings:
  - Reliability
  - Safety

Thank you for your attention!