

Demand Modelling with EMME/2

Presented by

Isabelle Constantin, INRO Consultants Inc.

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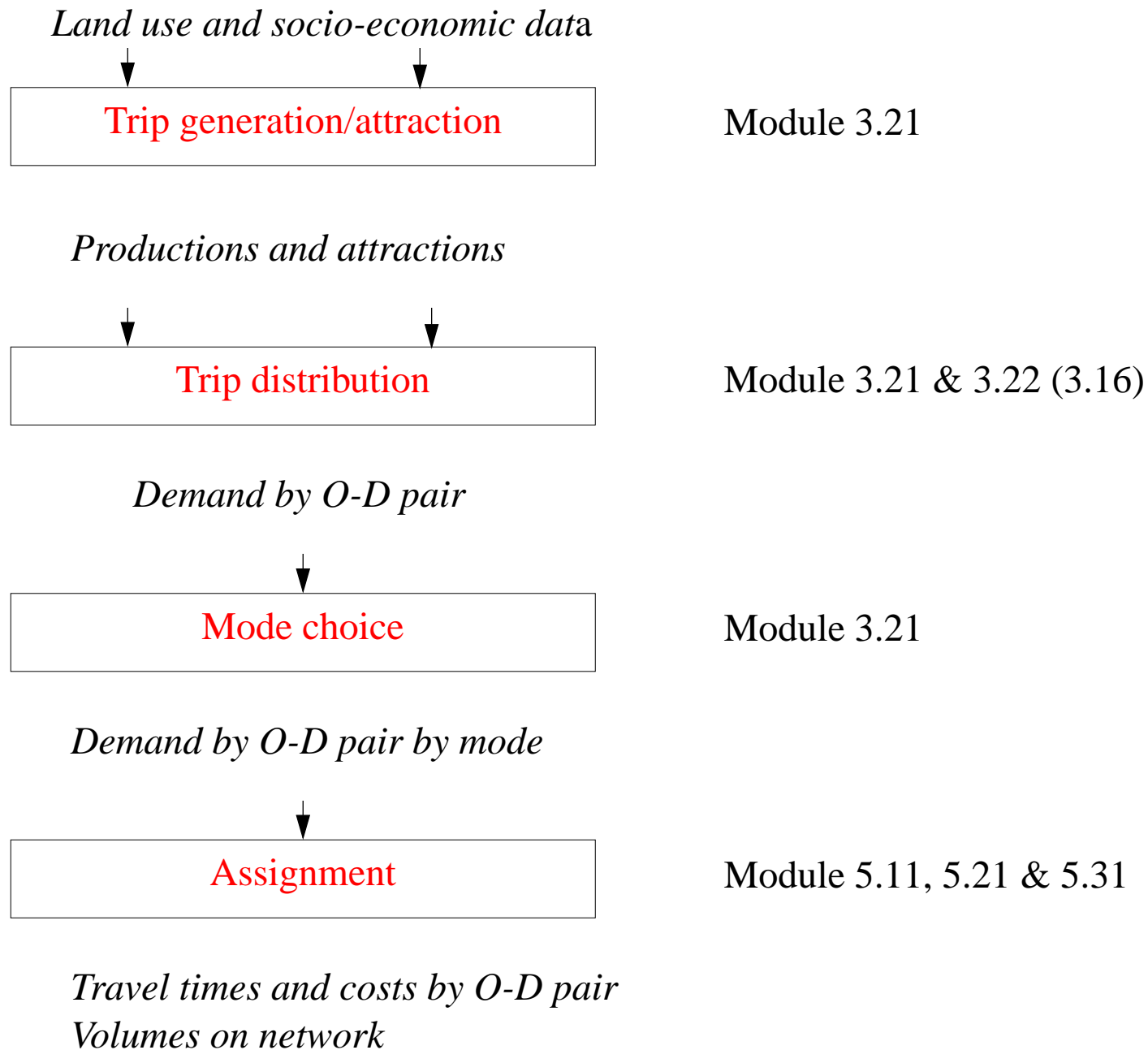
Tools Available in EMME/2

Instead of specific demand models, EMME/2 provides **tools**:

- Matrix manipulation tools
 - matrix **calculations**
 - matrix **balancing**
 - **matrix product** like operations
- Assignment procedures
 - equilibrium **auto assignment** (multiclass, generalized cost)
 - multi-path **transit assignment**
- **Macro** language

⇒ Allows implementing a **wide variety** of demand models

Implementing a 4-Step Model with EMME/2



Trip Generation & Attraction Models

- **Zone data:**
 - **population characteristics:** places of residence, dwelling types, age, ...
 - **economic activities:** number of jobs, type of industry, office surface area, ...
- **Models:**
 - attraction and production coefficients** obtained by calibrating models (econometric models, category analysis, ...)
- Examples:

$$O_p = F(\text{household_size}_p, \text{household_income}_p)$$

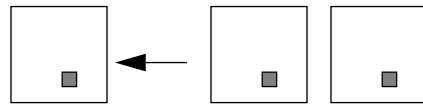
"**produc**" = "homes" * 2.9 for suburban zones
 = "homes" * 1.7 for center zones

$$D_q = F(\text{retail_employment}_q, \text{other_employment}_q, \text{households}_q)$$

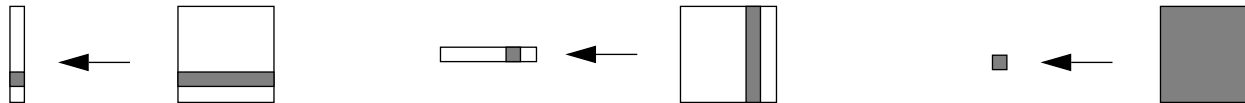
"**attrac**" = "indust" * 350 + "office" * 40 for industrial zones
 = "indust" * 75 + "office" * 80 for business zones

Matrix Calculations (Module 3.21)

- Evaluation of **algebraic expressions** involving matrices or zone data
- Calculation performed **element by element**: each element r_{pq} of result matrix is obtained from the elements pq of the operand matrices



- **Aggregation** performed for vector or scalar results:



- Useful for **calculations performed for origins and/or destinations**, including **many kinds of demand models**

Trip Distribution Models

- Aim: **reproducing productions and attractions** (observed or predicted)
- Different input matrices yield different models. Examples:

Growth model (also called **scaling** or **Fratar**)

- starts with an existing O-D matrix

Gravity model

- minimize total travel time or cost.
- starts with a function of the travel time or cost $f_{pq} = F(u_{pq})$
- sometimes specified as a **look-up table**

Two-Dimensional Matrix Balancing (Module 3.22)

- Aim: determine an origin-destination matrix, g_{pq} , satisfying:

$$\sum g_{pq} = O_p \quad \text{for each } p \quad (\text{productions are respected})$$

$$\sum_p g_{pq} = D_q \quad \text{for each } q \quad (\text{attractions are respected})$$

$$g_{pq} \geq 0 \quad \text{for each } (p, q)$$

- The resulting **balanced matrix** can be expressed as $g_{pq} = \alpha_p \beta_q f_{pq}$ where: α_p and β_q are the **balancing coefficients**.
- Useful for **implementing distribution models**:
different input matrices $f_{pq} \Rightarrow$ different models.

2-D Matrix Balancing Algorithm

Step 0. Initialization

$$l = 0$$

$$\alpha_p^{(0)} = 1 \quad \text{for each } p$$

$$\beta_q^{(0)} = 1 \quad \text{for each } q$$

Step 1. Balancing rows

$$\alpha_p^{(l+1)} = \frac{O_p}{\sum_q \beta_q^{(l)} f_{pq}} \quad \text{for each } p$$

Step 2. Balancing columns

$$\beta_q^{(l+1)} = \frac{D_q}{\sum_p \alpha_p^{(l+1)} f_{pq}} \quad \text{for each } q$$

Step 3. Stopping test

$$l = l + 1$$

$$\text{If } \max_p \left| \alpha_p^{(l)} \cdot \sum_q \beta_q^{(l)} f_{pq} - O_p \right| \leq \varepsilon \text{ stop}$$

else return to **Step 1**.

Note: If $\sum_p O_p = \sum_q D_q$, this algorithm converges (in most cases)

Example: Gravity Model Based on Look-up Table

- Prepare input matrix according to look-up table

| Interval | Travel time | F-factor |
|----------|-------------|----------|
| 1 | 0-5 | 0.15 |
| 2 | 5-10 | 0.22 |
| 3 | 10-15 | 0.27 |
| 4 | 15-20 | 0.24 |
| ... | | |

Example of EMME/2 macro (for module 3.21)

```
~g1=.15
~g2=.22
~g3=.27
~g4=.24
...
"gravit" = get(int("time"/5)+1)*"ffact"
```

- 2-dimensional balancing (module 3.22)
 - specify productions and attractions
 - input matrix: **"gravit"**

Example: Entropy Model

- Special case of gravity model:
 - maximize entropy (small values everywhere instead of large values and 0's)
 - minimize total travel time (or cost) $\Rightarrow f_{pq} = e^{-\theta u_{pq}}$
 - u_{pq} = estimated cost
 - θ = constant to calibrate
- Prepare input matrix (module 3.21)
 - `"entrop" = exp(-.059*"autime")*("obsdem">0)`
- 2-dimensional balancing (module 3.22)
 - specify productions and attractions
 - input matrix: `"entrop"`

2-D Balancing With Multiple Production Types: BALMPROD

- One attraction vector D_q and **up to 4 production vectors** Oi_p and corresponding friction matrices Ci_{pq} can be specified:

$$\begin{array}{ccccccccccc}
 D_q & C1_{pq} & O1_p & C2_{pq} & O2_p & C3_{pq} & O3_p & C4_{pq} & O4_p & & \\
 \text{[Diagram: A horizontal bar above a square, followed by a square, a vertical bar, a square, a vertical bar, a square, a vertical bar, a square, a vertical bar, and a square.] } & & & & & & & & & & \\
 \text{Must satisfy } \sum_q D_q & = & \sum_p O1_p & + & \sum_p O2_p & + & \sum_p O3_p & + & \sum_p O4_p & &
 \end{array}$$

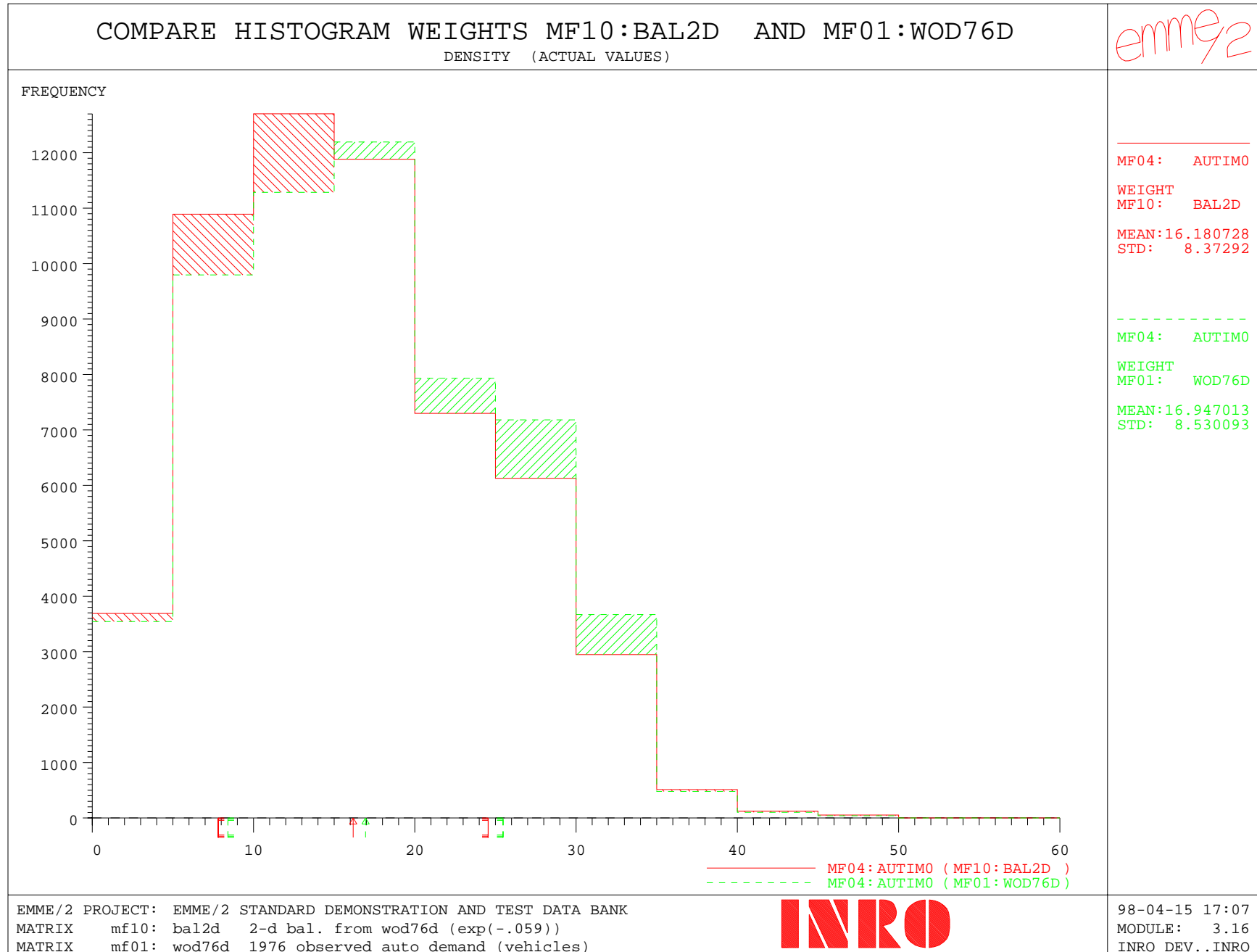
- Results:
 - balancing coefficients by destination: **md91**
 - balancing coefficients by origin for production type i : **mo9i**
 - the balanced matrix for production type i can then be computed as:
mo9i * md91 * mf "Cpqi"
- Example:


```
~<balmprod md"attrac" mf"fwork" mo"pwork" mf"fstud" mo"pstud" mf"fshop" mo"pshop"
```

The resulting matrix for *shopping trips* can then be computed as

```
mo93*md91*mf"fshop"
```

Comparison of Travel Time Distribution (5-min Intervals)



Three-Dimensional Matrix Balancing (Module 3.22)

- Aim: determine an origin-destination matrix, g_{pq} , satisfying:

$$\sum_q g_{pq} = O_p \quad \text{for each } p \quad (\text{productions are respected})$$

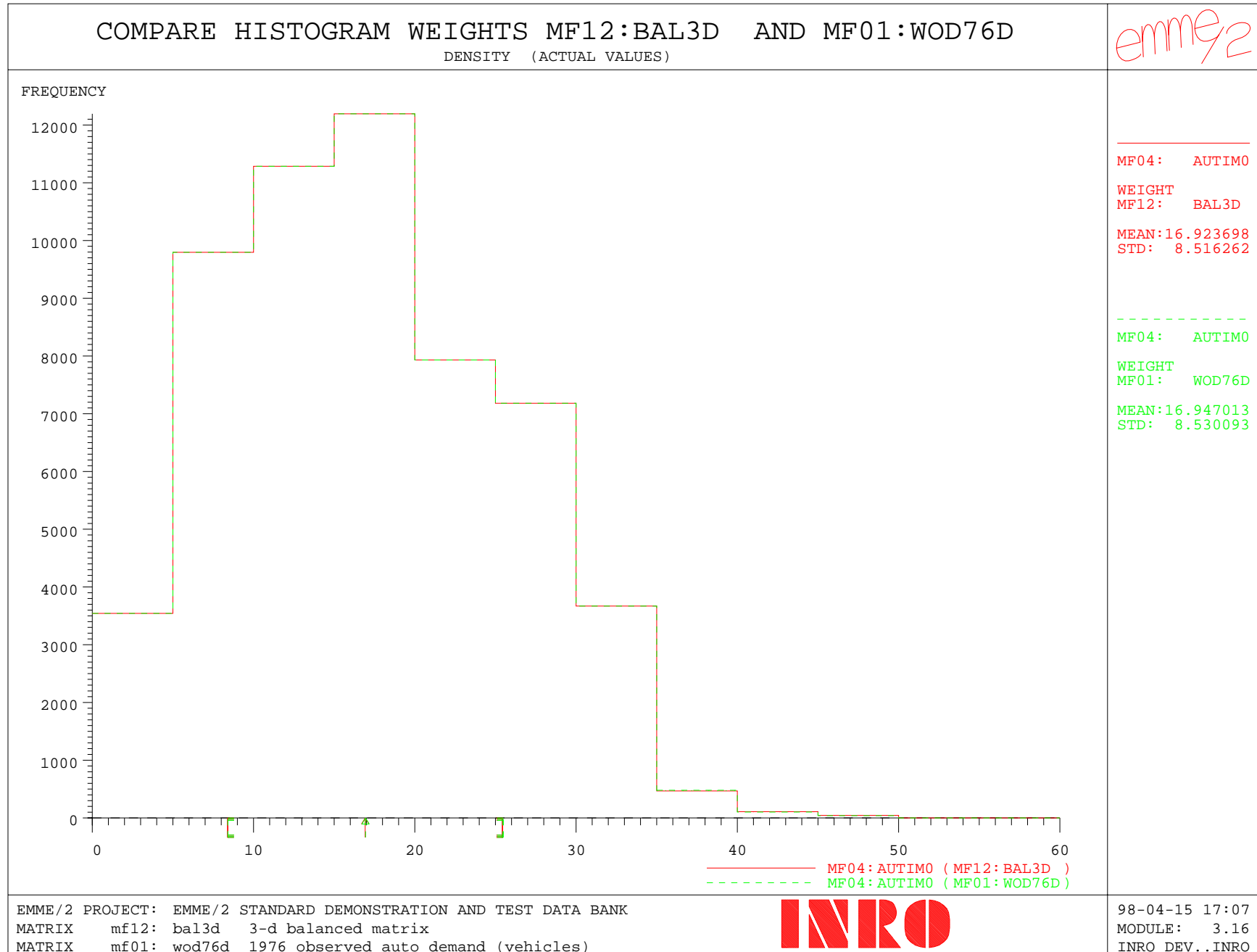
$$\sum_p g_{pq} = D_q \quad \text{for each } q \quad (\text{attractions are respected})$$

$$g_{pq} \geq 0 \quad \text{for each } (p, q)$$

$$\sum_{(p, q) | k_{pq} = k} g_{pq} = F_k \quad \text{for each } k \quad (\text{total respected for each category})$$

- Requires:
 - an index matrix specifying to which category each O-D pair belongs
 - **totals for each category** (district-to-district, travel time interval, ...)
- Algorithm contains an additional step to balance the third dimension.
- The resulting **balanced matrix** can be expressed as $g_{pq} = \alpha_p \beta_q \gamma_{k_{pq}} f_{pq}$ where α_p , β_q and $\gamma_{k_{pq}}$ are the **balancing coefficients**.
- Useful for **refining distribution models**, by including an additional constraint.

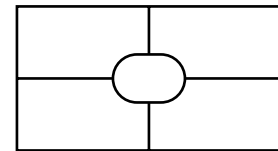
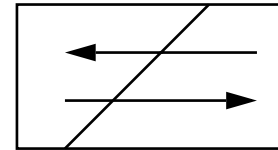
Comparison of Travel Time Distribution (5-min Intervals)



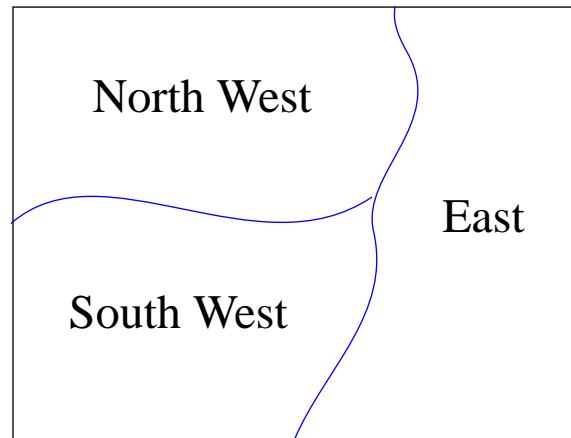
Refining a Trip Distribution Model

Include an additional constraint based on:

- Travel time distribution
- Screen lines
 - 3 categories of trips:
 - trips crossing in each direction
 - trips remaining on the same side
 - screen line counts must include all crossings
- Boundaries
 - physical and/or psychological (districts, rivers, etc.)
 - may make some minutes appear longer than others
 - define N sets of zones $\Rightarrow N^2$ categories of trips
 - set of totals must be reliable and consistent



Example: Reproducing Aggregated Totals



| From | To | Observed | 2-D model |
|-----------|-----------|----------|-----------|
| <i>NW</i> | <i>NW</i> | 16774 | 17694 |
| <i>NW</i> | <i>E</i> | 2530 | 2142 |
| <i>NW</i> | <i>SW</i> | 3513 | 2981 |
| <i>E</i> | <i>NW</i> | 9942 | 9499 |
| <i>E</i> | <i>E</i> | 6389 | 6741 |
| <i>E</i> | <i>SW</i> | 2921 | 3012 |
| <i>SW</i> | <i>NW</i> | 8402 | 7924 |
| <i>SW</i> | <i>E</i> | 1359 | 1393 |
| <i>SW</i> | <i>SW</i> | 4389 | 4830 |

Example: Reproducing Aggregated Totals (Continued)

- Number the categories. For example:

| | NW | E | SW |
|----|----|----|----|
| NW | 11 | 12 | 13 |
| E | 21 | 22 | 23 |
| SW | 31 | 32 | 33 |

- Compute the index matrix (module 3.21)

$$\text{"index"} = 10 * \text{gw}(\text{p}) + \text{gw}(\text{q})$$

- 3-dimensional balancing (module 3.22)
 - input matrix: **"entrop"**
 - 3-d index matrix: **"index"**
 - totals for third dimension: **observed totals**

- Aggregated totals for 3-D balanced matrix correspond exactly to those observed

| origin | destin | observ | 3dbal |
|--------|--------|--------|-------|
| gw01 | gw01 | 16774 | 16774 |
| gw01 | gw02 | 2530 | 2530 |
| gw01 | gw03 | 3513 | 3513 |
| gw02 | gw01 | 9942 | 9942 |
| gw02 | gw02 | 6389 | 6389 |
| gw02 | gw03 | 2921 | 2921 |
| gw03 | gw01 | 8402 | 8402 |
| gw03 | gw02 | 1359 | 1359 |
| gw03 | gw03 | 4389 | 4389 |

3-D Balancing with Group-to-Group Totals: BALANCE3

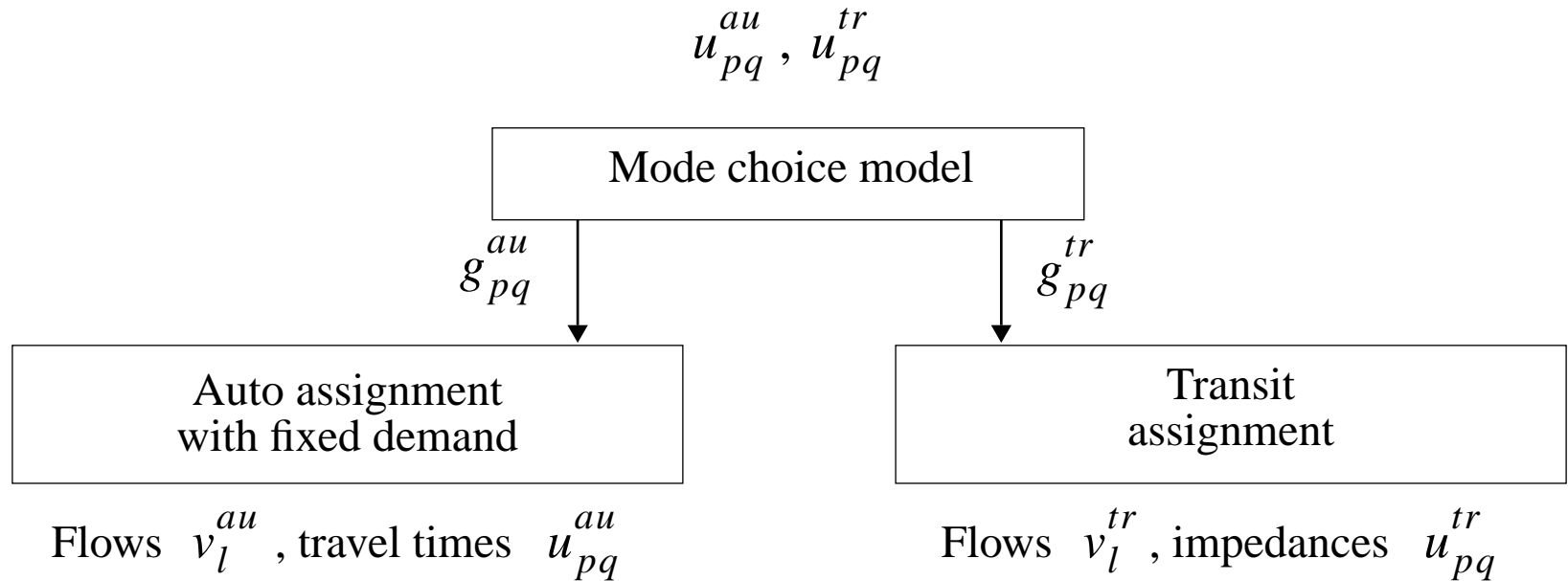
- Performs a 3-D matrix balancing using module 3.21 only
 - ⇒ number of 3-D constraints not limited to a maximum of 250, but procedure less efficient than module 3.22.
- 3-D constraints are the group-to-group totals of an aggregated matrix.
- Usage: `~<balance3 Cpq Op Dq Rpq gx Ap Bq Gpq`
 - *Cpq* contains the desired input matrix (entropy, etc.)
 - *Rpq* contains the 3rd dimension group-to-group totals (according to **gx**) replicated in all corresponding O-D pairs

Example:

```
~<balance3 "entrop" "produc" "attrac" "dtod" w mo97 md97 mf15
```

- The procedure might not converge if:
 - the group subtotals in matrix *Rpq* do not match the corresponding production and attraction subtotals in *Op* and *Dq*
 - ...

Mode Choice Models



- Resulting u_{pq}^{au}, u_{pq}^{tr} not necessarily consistent with those used as input to mode choice model \Rightarrow various **feedback** techniques
- **Many time and cost components needed for each O-D pair:**
 - drive-alone: time, distance, toll, parking cost, ...
 - car pool (HOV): time saving, ...
 - transit with walk access: walk, in-vehicle, waiting, fare, ...
 - transit with auto access: auto time, transit times, parking cost, ...

Auto Assignment (Modules 5.11 & 5.21)

Standard assignment

- **Equilibrium** assignment (“all-or-nothing” assignment \Rightarrow iteration 0)
- Based on **time** or **generalized cost**
- Output matrix: *travel time* or *generalized cost*

Additional options assignment

- Computation of other **O-D attributes** such as *distance*, *toll*, ...
- Typical set-up:

| | |
|------------------------------|------------------------------------|
| <i>Additional attribute:</i> | contents depends on desired result |
| <i>Path operator:</i> | + |
| <i>Threshold:</i> | none |
| <i>O-D attribute:</i> | path attribute |

Transit Assignment (Modules 5.11 & 5.31)

Standard assignment

- **Multi-path** assignment based on *optimal strategies*
- Output matrices:
 - *transit time* (weighted sum)
 - *time components* (unweighted, for 1 or more mode)
 - *average number of boardings* (for 1 or more mode)

Additional options assignment

- Computation of other **O-D attributes** such as *distance, fare, ...*
- Typical set-up:

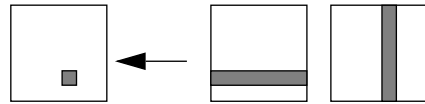
| | |
|-------------------------------------|---|
| <i>Additional attributes:</i> | which attribute and its contents depend on desired result |
| <i>Path operator:</i> | + |
| <i>Sub-strategy comb. operator:</i> | average |
| <i>Retained part of strategies:</i> | complete strategies |
| <i>Threshold:</i> | none |
| <i>Matrix saved:</i> | additional strategy attribute |

Modelling Park-and-Ride: Best Total Time

- Aim:
 - for each O-D pair, find the “best” parking lot to use for P&R, that is, the one for which $a_{pk} + t_{kq} + c_k$ is minimum
 - obtain **corresponding travel time matrices** for car and transit
- Required data:
 - network where **parking lots are coded as centroids** (\Rightarrow zones)
 - auto time (or impedance) matrix
 - transit time (or impedance) matrix
 - parking cost vector (optional)
- Desired results
 - matrix containing the **zone number of the parking lot** with
$$\text{Min } (a_{pk} + t_{kq} + c_k)$$
 - two **impedance matrices**:
 - **auto** impedance from origin to best parking lot
 - **transit** impedance from best parking lot to destination

Matrix Convolutions (Module 3.23)

- Matrix product like operations
- Each element r_{pq} of result matrix is obtained from the elements of row p of the 1st operand matrix and column q of the 2nd operand matrix



- Useful for **calculations involving intermediate zones**, including more complex demand models

From Matrix Product to Matrix Convolution

Matrix product

- Each element of row p of A is **multiplied** by the corresponding element of column q of B , and the intermediate results are **summed**

- $$r_{pq} = \sum_k a_{pk} \times b_{kq}$$

Triple-index operation

- Different **combination** \otimes and **contraction** \oplus operators may be used

- $$r_{pq} = \bigoplus_k a_{pk} \otimes b_{kq}$$

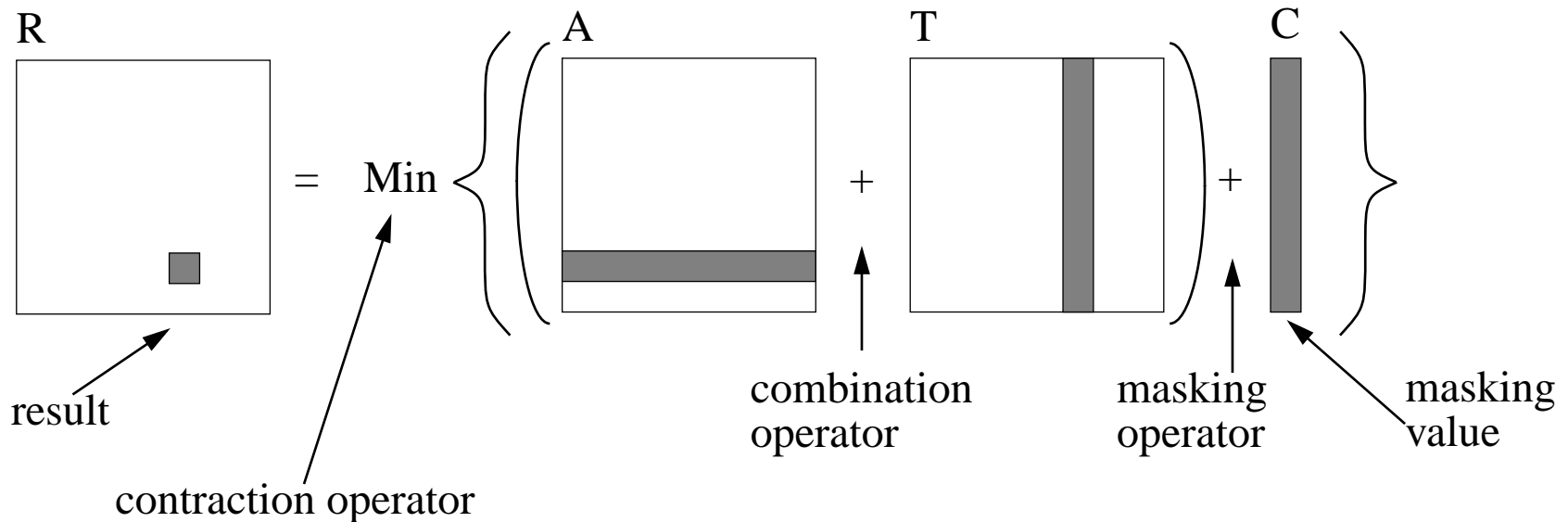
Convolution

- **Further processing** may be performed on $a_{pk} \otimes b_{kq}$, before contraction operator \oplus is applied to the intermediate results

Finding the Best Parking Lot (Convolution)

Minimum total time:

$$r_{pq} = \text{Min}_k \{ (a_{pk} + t_{kq}) + c_k \}$$



Corresponding parking lot: $l_{pq} = \text{Argmin}_k \{ (a_{pk} + t_{kq}) + c_k \}$

Extracting the Auto and Transit Travel Times

- Aim: **extract the impedance** for each leg of the trips
 Upk for leg 1
 Ukq for leg 2
 based on the *selected parking lot*
- Macro: **~<legimped leg Upq Kpq Lpq**
 leg 1 or 2
 Upq impedance matrix to extract leg impedances from
 Kpq index matrix containing intermediate zone numbers
 Lpq **resulting first or second leg impedance matrix**

Example:

```
~<legimped 1 autim0 bstlot mf11  
~<legimped 2 trtim0 bstlot mf12
```

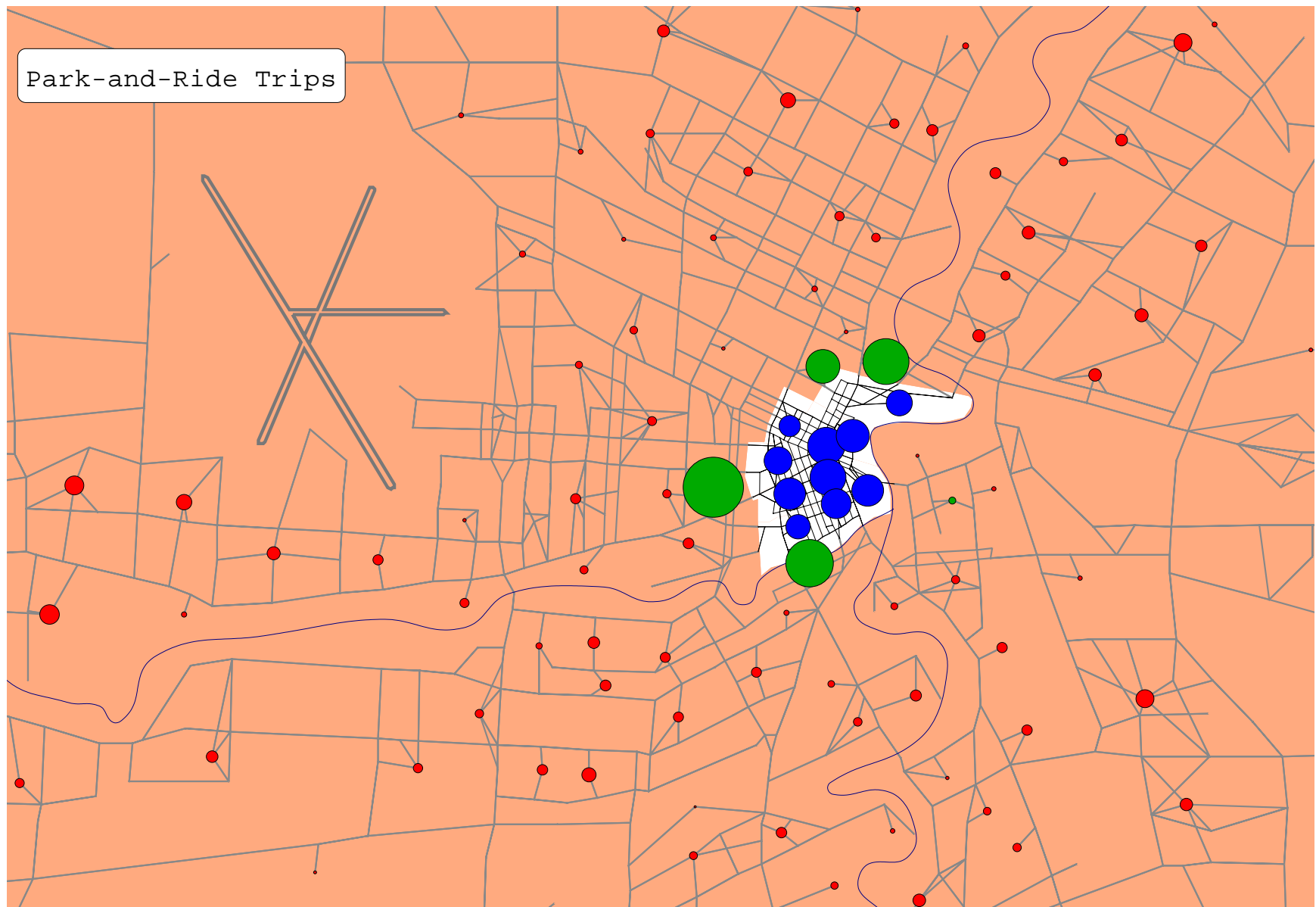
Splitting the P&R Demand into Auto and Transit Demand

- Aim: **split** the park-and-ride **demand** for the 2 legs of the trips
 G_{pk} for leg 1
 G_{kq} for leg 2
 based on the *selected parking lot*
- Macro: `~<splitmat G_{pq} K_{pq} G_{pk} G_{kq}`
 - G_{pq} demand matrix to be split
 - K_{pq} index matrix containing intermediate zone numbers
 - G_{pk} demand from the origins to the intermediate zones
 - G_{kq} demand from the intermediate zones to the destinations

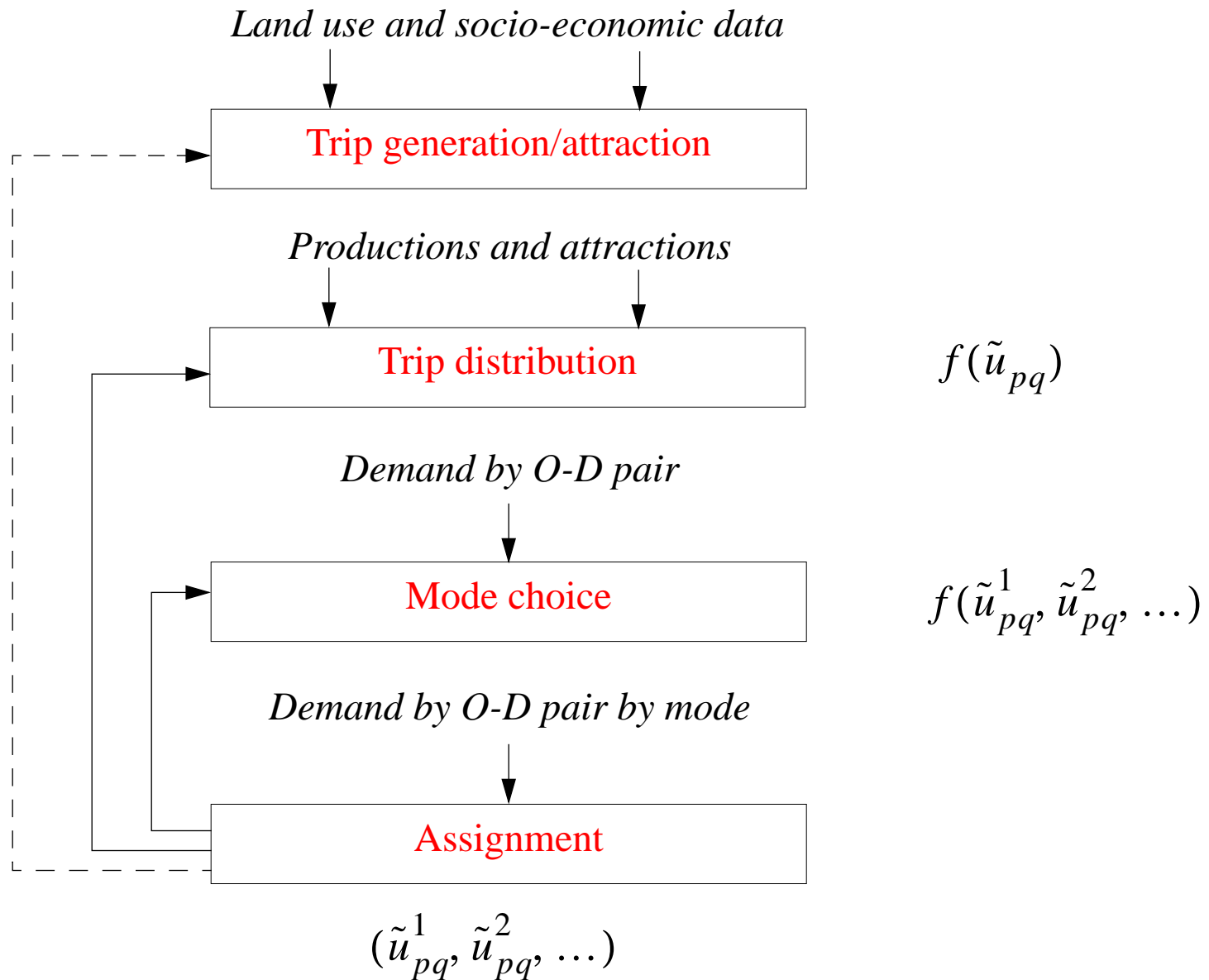
Example:

```
~<splitmat prdem prlot mf9 mf10
```

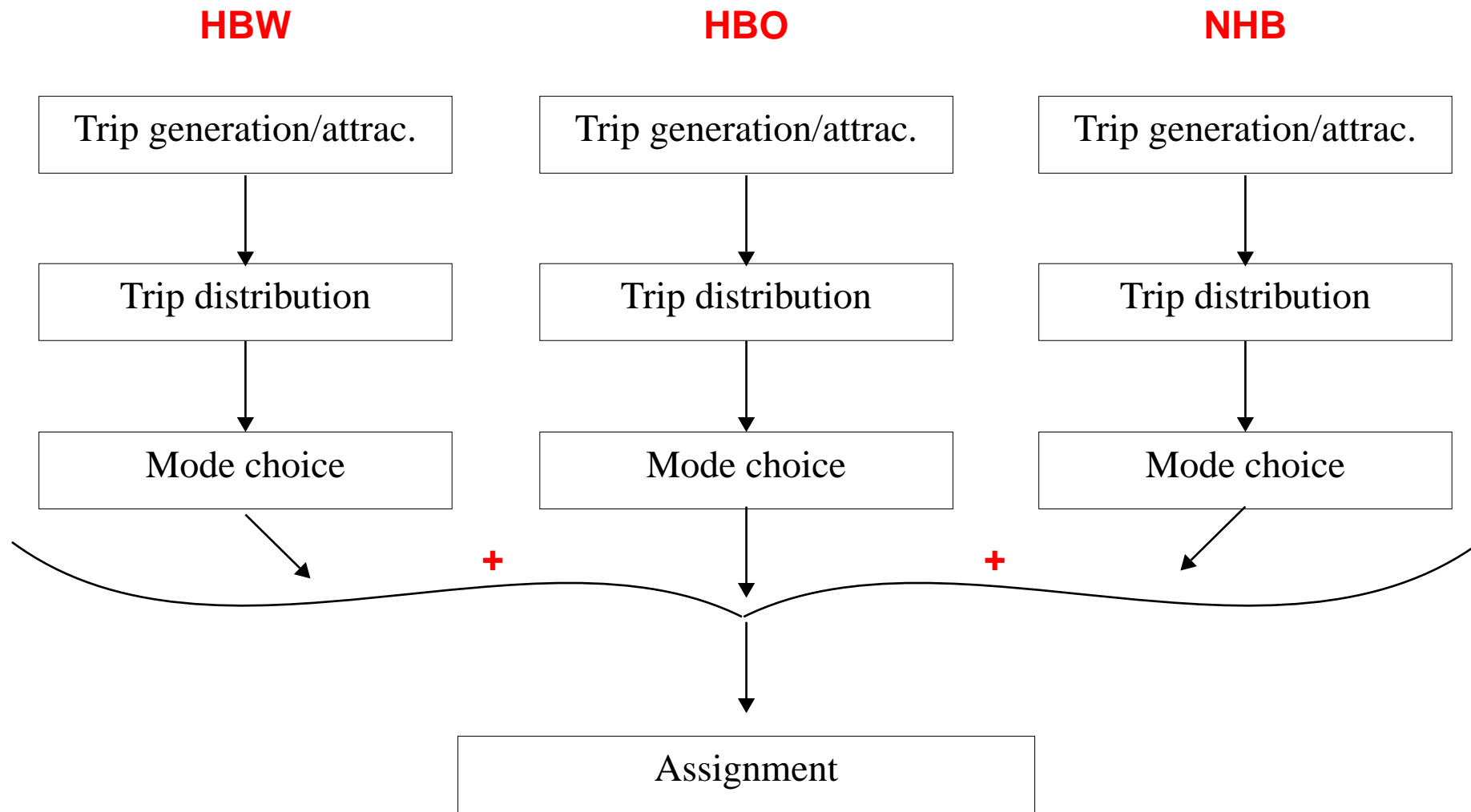
Park-and-Ride Trips



Feedback in a 4-Step Model



4-Step Model with Multiple Trip Purposes



Conclusions

Flexible framework for demand modelling!

⇒ matrix manipulation tools

⇒ assignment procedures

⇒ macros available on the web
www.spiess.ch

Share experience!

⇒ papers available on the web
www.inro.ca

⇒ list server for EMME/2 users
www.egroups.com/group/emme2users