A Multi-Class Network Equilibrium Model for Mixed Traffic of Cars and Trucks: Application to The SCAG Travel Demand Model

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Problem Description

- A multi-class network equilibrium model;
- Volume delay functions depend on the mix of trucks and cars as well as the link length and the slope of the link ;
- As a consequence, the cost functions are nonlinear, non-smooth and asymmetric
- A large scale problem (6 classes of traffic, 3217 zones and 99867 links in SCAG)

VDF and PCE

- VDF (volume delay function): Each class has its own travel time depending on link volume and free flow speed on the link
- PCE (passenger car equivalents): truck volumes are converted to car volumes. The conversion depends on
    - mix of traffic (congestion factor)
    - percentage of each class of traffic on the link
    - slope and length of the link
    - by each class

Heavy Duty Truck PCE Values $PCE_a^m(p_a^m, l_a, g_a)$

p_a^m	l_a	*m=4*			*m=5*			M=6		
		g_a								
		0-2	3-4	>4	0-2	3-4	>4	0-2	3-4	>4
0-5	≤ 1	2.0	4.2	6.4	3.4	6.9	8.8	4.3	8.0	11.3
	>1	2.0	5.5	7.5	5.2	8.4	10.7	6.7	10.5	13.5
5-10	≤ 1	2.0	3.4	4.8	3.1	5.0	6.4	3.5	5.8	8.8
	>1	2.0	4.2	5.3	3.9	5.9	7.8	4.8	7.8	13.5
>10	≤ 1	2.0	3.3	4.1	2.8	4.5	6.0	3.2	5.1	8.3
	>1	2.0	3.5	5.0	3.7	5.3	7.5	4.0	7.5	12.5

p_a^m -- Truck percentage of class m on link a

l_a -- link length

g_a -- link grade

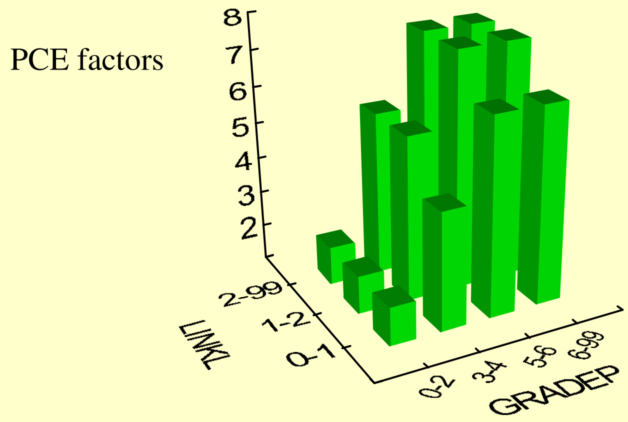
adj_a^m values m=4,5,6

vc_a	p_a^4			p_a^5			p_a^6		
	0-5	5-10	>10	0-5	5-10	>10	0-5	5-10	>10
<0.5	0.60	0.66	0.90	0.66	0.77	0.93	0.90	0.77	0.93
0.5-1.0	0.77	0.89	1.15	0.89	1.01	1.20	1.15	1.01	1.20
1.0-1.5	1.10	1.20	1.30	1.20	1.25	1.34	1.30	1.25	1.34
1.5-2.0	1.00	1.05	1.22	1.05	1.22	1.25	1.22	1.22	1.25
>2.0	1.19	0.66	1.26	1.05	1.24	1.29	1.26	1.24	1.29

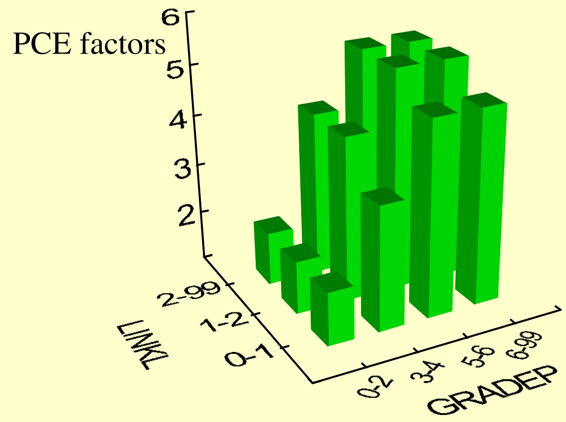
p_a^m -- Truck percentage of class m on link a

vc_a -- volume/capacity

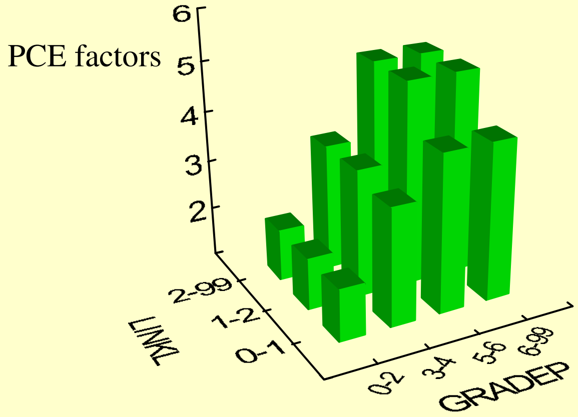
PCE FACTORS (LH Truck Percent)



Truck percent 0 - 5



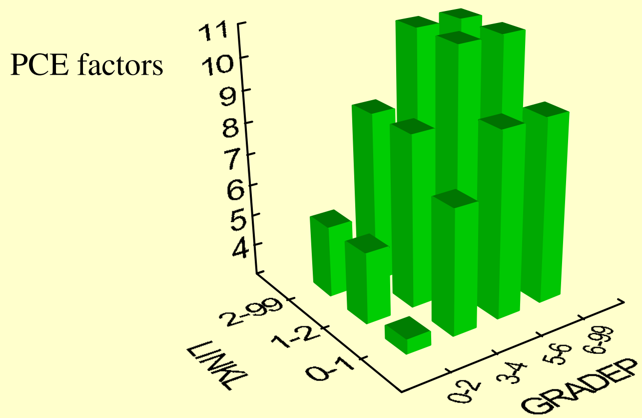
Truck percent 5 -10



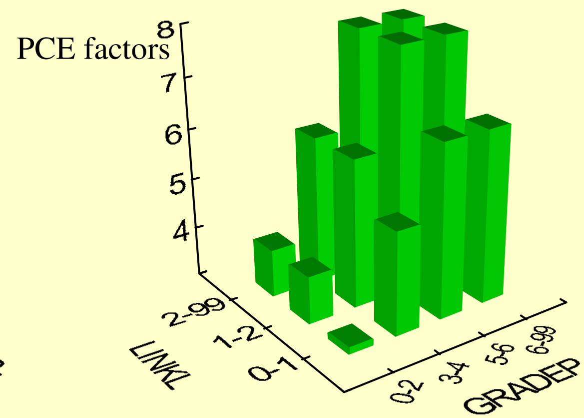
Truck percent >10

LEGEND
LINKL - link length
GRADEP - grade percent

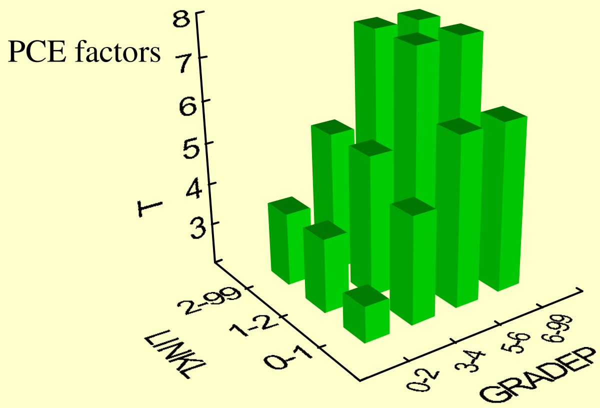
PCE FACTORS (MH Truck Percent)



Truck percent 0 - 5



Truck percent 5 -10



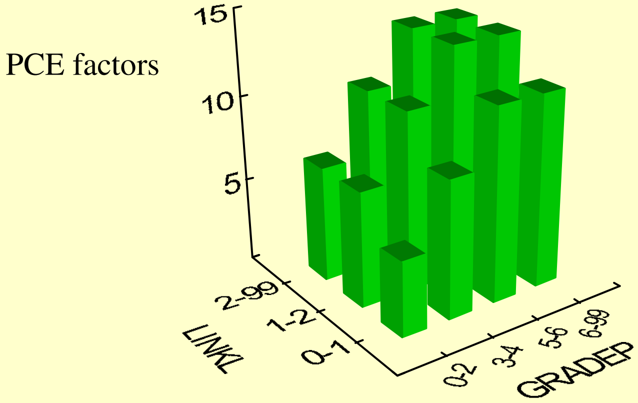
Truck percent >10

LEGEND

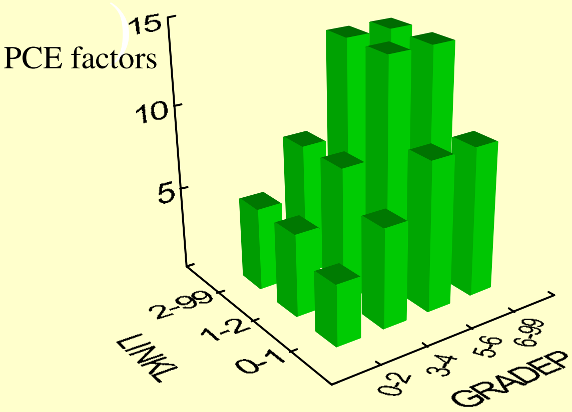
LINKL - link length

GRADEP - grade percent

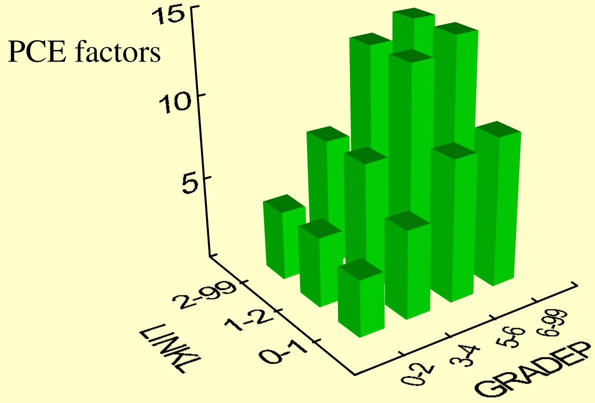
PCE FACTORS (HH Truck Percent)



Truck percent 0 - 5



Truck percent 5 -10



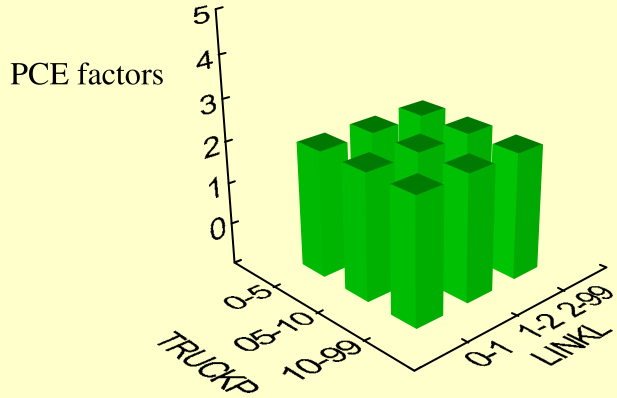
Truck percent >10

LEGEND

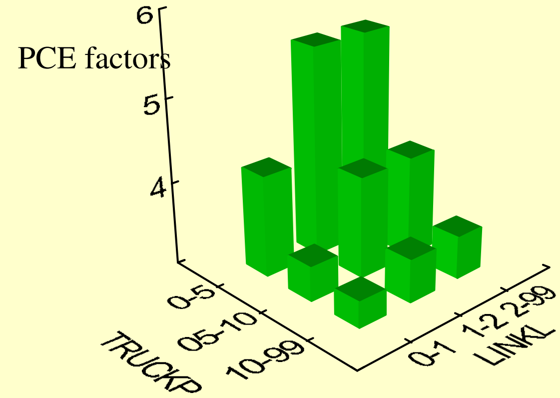
LINKL - link length

GRADEP - grade percent

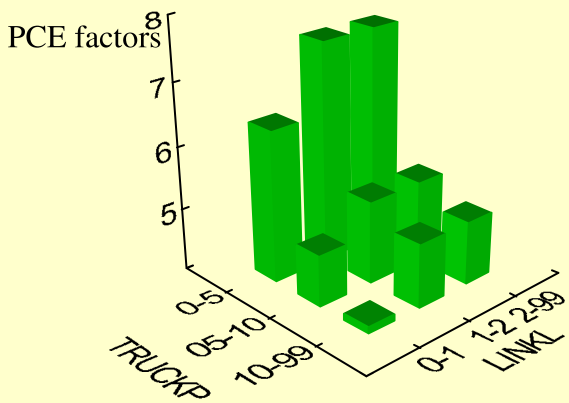
PCE FACTORS (LH Percent Grade)



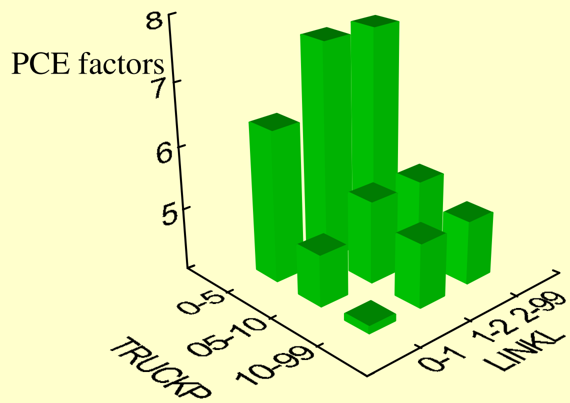
percent grade 0 - 2



percent grade 3 -4



percent grade 5 - 6

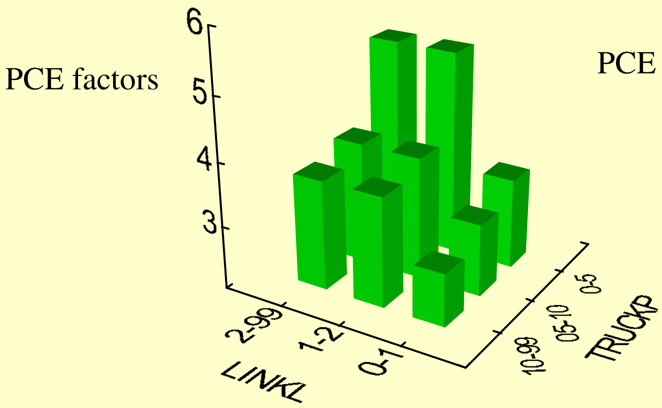


percent grade > 6

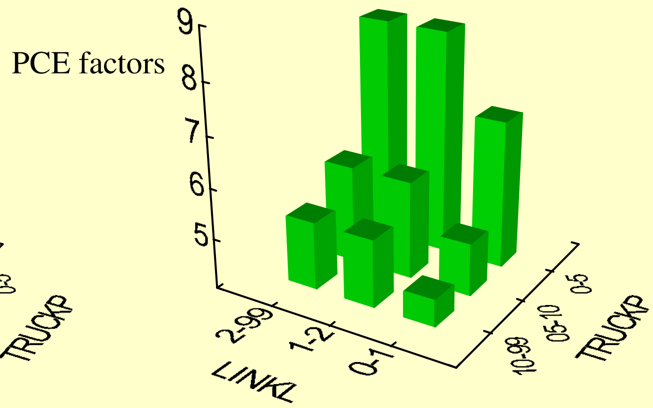
LEGEND

LINKL - link length
TRUCKP - truck percent

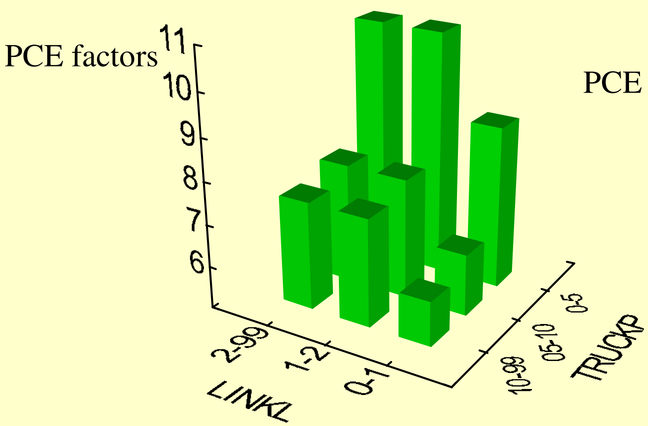
PCE FACTORS (MH Percent Grade)



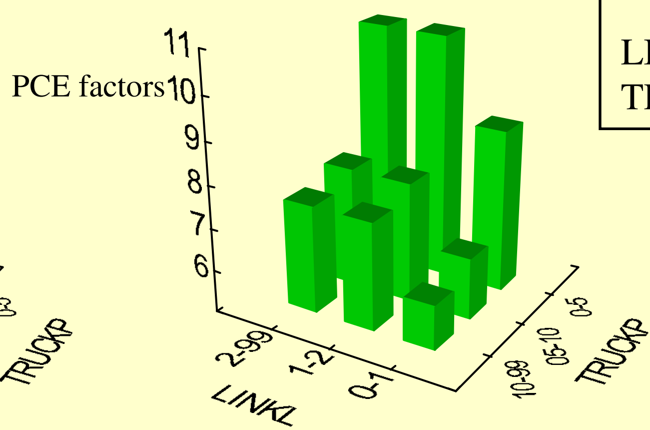
percent grade 0 - 2



percent grade 3 -4



percent grade 5 - 6

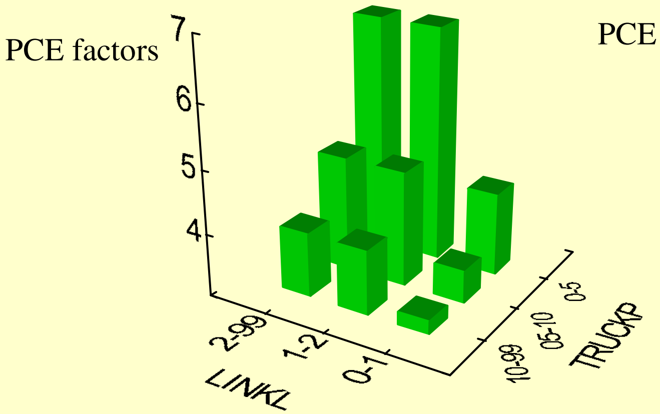


percent grade > 6

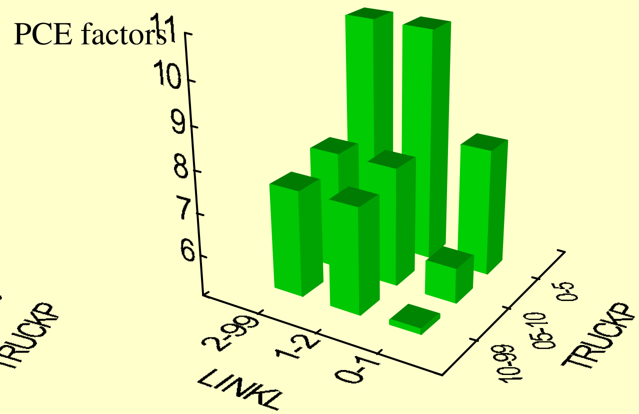
LEGEND

LINKL - link length
TRUCKP - truck percent

PCE FACTORS (HH Percent Grade)



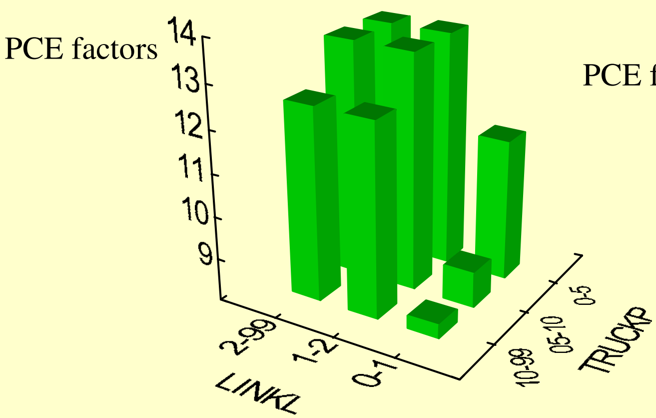
percent grade 0 - 2



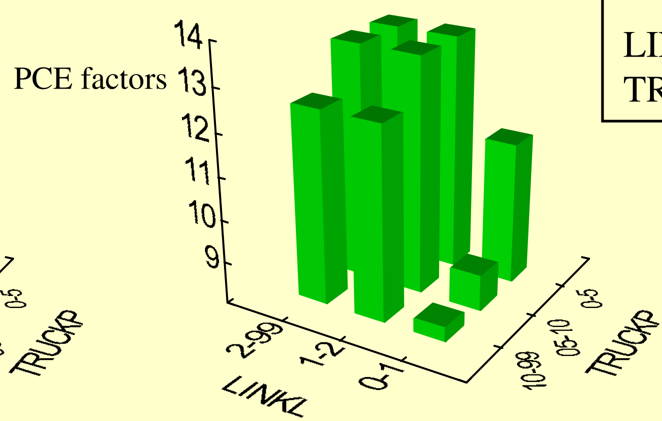
percent grade 3 -4

LEGEND

LINKL - link length
TRUCKP - truck percent

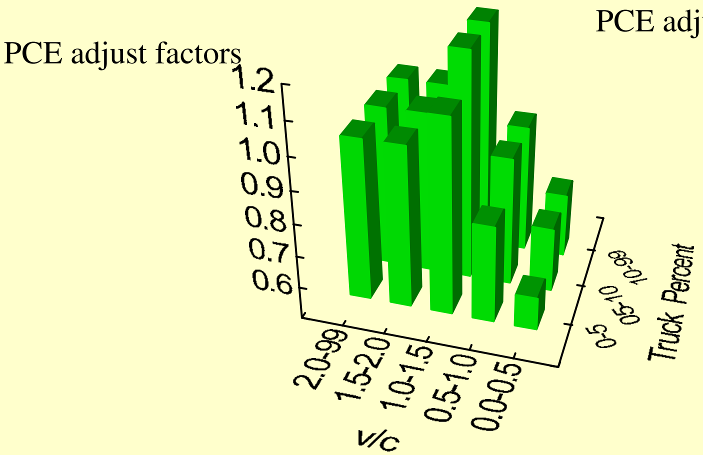


percent grade 5 - 6

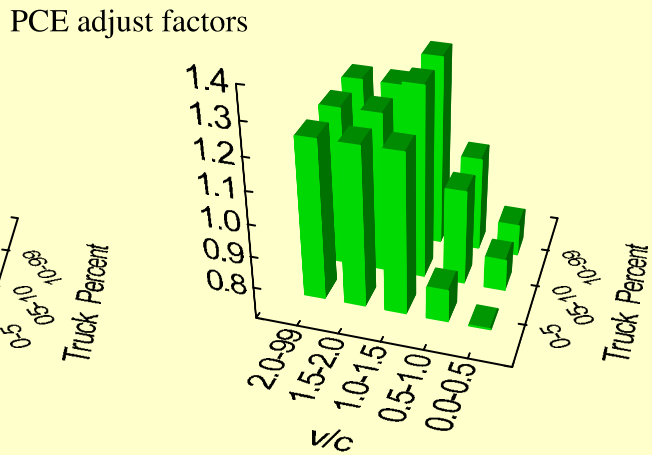


percent grade > 6

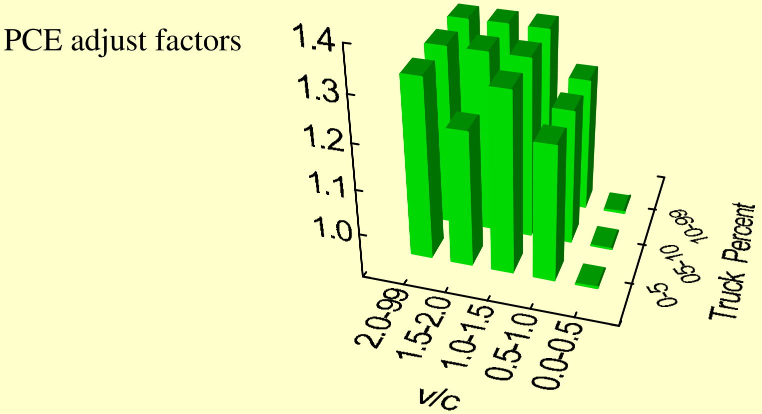
Congestion PCE adjust Factors



Light heavy duty truck



Medium heavy duty truck



Heavy heavy duty truck

LEGEND

v/c - volume/capacity
truck percent

Literature

- The literature that does not offer much help in solving such a model. Some asymmetric models are considered by
  - Marcotte and Zhu (1996)
  - Magnanti and Perakis (1997)

  Use an LP based operator and a projection operator respectively and prove the convergence of these algorithms under certain condition.

Model Formulation: Notation

Sets and indices used

A	:link set of base network in period t	a	:link index of base network
W	: total OD pairs	w	: OD pair index
R_{ij}^{m}	: set of routes for pair (ij), class m	r	: path index
M	: class set	m	: class index

Model Formulation: Notation

The variables include:

h_r^m : The path flow of class m on the route r.

f_a^m : The total vehicle link flow of class m on link a .

C_r^m : The path travel time of class m on route r.

v_a^m : Link flow in PCE of class m on link a

$v_a = \sum_m v_a^m$: Link flow in PCE of all classes on link a

$c_a(v_a)$: travel time on link a for link flow v_a

Total link flow in PCE

- The PCE flow may be expressed as a nonlinear function:

$$v_a^m = F(f_a^m, p_a^m, vc_a, \kappa_a, adj_a^m, l_a, g_a \,|\, \forall m \in M)$$

where

p_a^m is the percentage of the link flow of class *m* on link *a*

$$p_a^m = f_a^m / \sum_{m' \in M} f_a^{m'},$$

vc_a is the link flow in PCE, v_a, over the capacity of link *a*, κ_a, given as

$$vc_a = v_a / \kappa_a,$$

κ_a is the link capacity in PCE on link *a*,

$adj_a^m(vc_a)$ is an adjustment factor of class *m* on link *a* based on vc_a,

l_a is the length on link *a*,

g_a is the grade of link *a*.

Total link flow in PCE

- In practice, a look-up table is used.

$$v_a^m = PCE_a^m(p_a^m, \kappa_a, adj_a^m(vc_a), l_a, g_a) \times f_a^m$$

where

$$PCE_a^m(p_a^m, \kappa_a, adj_a^m(vc_a), l_a, g_a)$$

is computed by using the SCAG coefficients

Mathematical Model

- The feasible region of the problem is defined as follows:

$$\sum_{r} h_r^m = T_{ij}^m, \quad \forall (ij), m \in M$$

Conservation of flow

$$h_r^m \geq 0, \forall m \in M, r \in R_{ij}^m$$

Non-negativity of the path flows

where $$f_a^m = \sum_{r \in R} h_r^m \delta_{ar}, \quad \forall\, a \in A, m \in M$$

δ_{ar} =1, if a is on route r

$$C_r^m = \sum_{a \in A} c_a^m (v_a) \delta_{ar}, \quad \forall\, m \in M, r \in R_{ij}^m$$

The path travel time

Variational inequality formulation

- Find *h** such that

$$\sum_{w \in W} \sum_{m \in M} \sum_{r \in R_w} C_r^m (h^*)(h_r - h_r^*) \geq 0, \forall h \in \Omega.$$

It is clear that the solution of the problem satisfies the following equilibrium conditions:

$$C_r^m \begin{cases} = u_w^m & if \quad h_r^m > 0 \\ \geq u_w^m & if \quad h_r^m = 0 \end{cases}, \forall m \in M, r \in R_w$$

where $u_w^m = \min_{r \in R_w} \{C_r^m\}$ is the minimum travel time for pair w of class m

(Wardrop user's optimal conditions)

Solution Algorithm

- LP-based operator of the solution algorithm

Step 0. Initialization. Start with $l = 0, f_a^{m,l} = 0$

Step. 1. Compute percentage of link flow, v/c ratio and link flow in PCE.

$$p_a^{m,l} = f_a^{m,l} / \sum_{m' \in M} f_a^{m',l}, \forall a \in A, m \in M$$

$$vc_a^l = v_a^l / \kappa_a, \forall a \in A$$

$$v_a^{m,l} = PCE_a^m(p_a^m, \kappa_a, adj_a^m(vc_a^l), l_a, g_a) \times f_a^{m,l}$$

Step 2. Computation of link cost

$$c_a^l(v_a^l), \forall a \in A$$

Solution Algorithm

Step 3. Computation of the shortest path flow problem for each class m:

$$\min \sum_m \sum_k C_k^{m,l} \bar{h}_k^{m,l}$$

$$s.t. \quad \sum_{r \in R_w^m} \bar{h}_r^{m,l} = T_w^m, \forall w \in W, m \in M$$

$$\bar{h}_r^{m,l} \geq 0, \forall r \in R_w^m, m \in M$$

Step 4. Computation of link flow $\bar{f}_a^{m,l} = \sum_{r \in R} \bar{h}_r^{m,l} \delta_{ar}, \forall a \in A, m \in M$

Step. 5. Computation of successive average (MSA)

$$f_a^{m,l+1} = f_a^{m,l} + (\bar{f}_a^{m,l} - f_a^{m,l}) / (l+1), \forall a \in A, m \in M$$

Step. 6. l=l+1. Go to step 1

Application

- An EMME/2 macro was developed for this application.
- The SCAG regional network was used.
- The macro was iteration zero (0) of the emme/2 multi-class assignment (shortest path) to compute the f_a^{m}'s .
- The volume/delay function is the classical BPR function:

$$c_a(v_a) = \beta_a(1 + 0.15 \times (v_a / \kappa_a)^4)$$

Six classes of demands

1. Passenger cars of one person
2. Passenger car of two person
3. Passenger car of three person +
4. Light-heavy duty trucks, 8500 to 14,000 GVW
5. Medium-heavy duty trucks, 14,000 to 30,000 GVW
6. Heavy-heavy duty trucks, over 30,000 GVW

Measures of Convergence

- M1: the relative difference between volume at iteration *l* and successive average volume at iteration *l*:

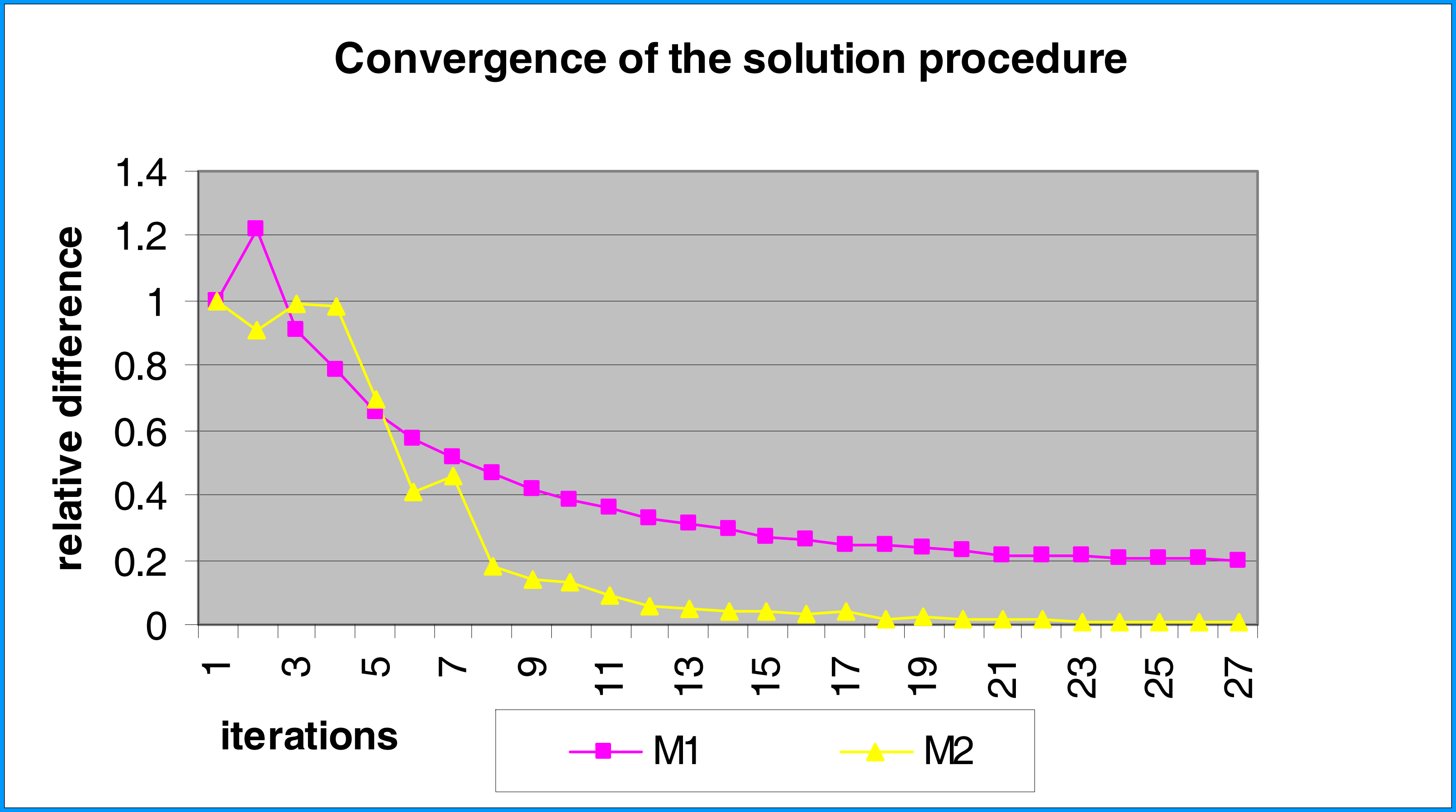
$$d^l = \sum_a \sum_m | \overline{f}_a^{m,l} - f_a^{m,l} | / (\sum_a \sum_m \overline{f}_a^{m,l})$$

- M2: the relative gap $rgap^l$ computed with the flow $\overline{f}_a^{m,l}$ which is the "all-or-nothing" assignment on shortest paths and the last flow $\overline{f}_a^{m,l-1}$ weighted by the current travel time $c_a(\overline{v}_a^l)$

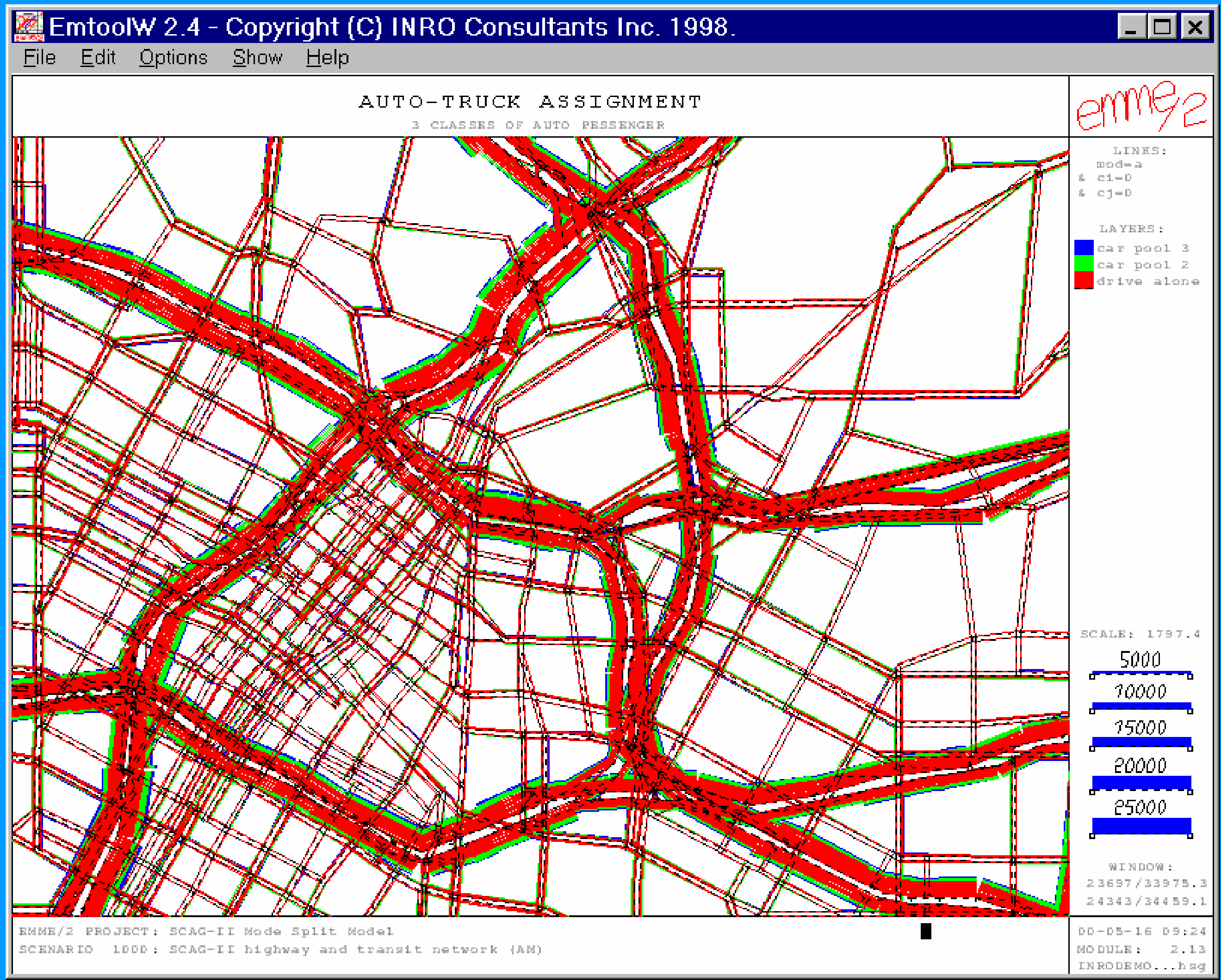
$$rgap^l = \sum_a \sum_m c_a(\overline{v}_a^l)(\overline{f}_a^{m,l-1} - \overline{f}_a^{m,l}) / (\sum_a \sum_m c_a(\overline{v}_a^l)\overline{f}_a^{m,l-1})$$

If $d^l \to 0$ *or* $rgap^l \to 0$ *as* $l \to \infty$

Convergence of the Procedure



Three classes of auto vehicles



Three classes of truck vehicles