The Equilibrium Assignment – What is a “Correct” Solution?

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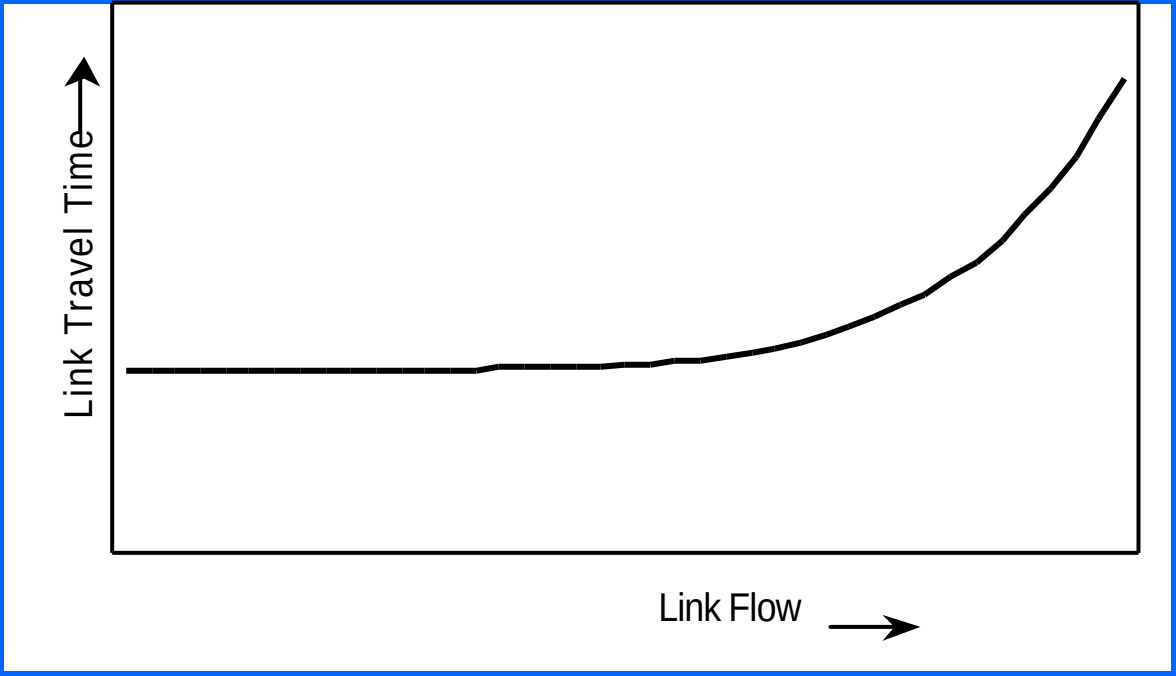
- Brief history of trip assignment
- Stopping criteria
- Results of analysis
- Conclusions & Recommendations

Assignment: A Brief History

- All-or-Nothing Assignments

Assignment: A Brief History

- **All-or-Nothing Assignments**

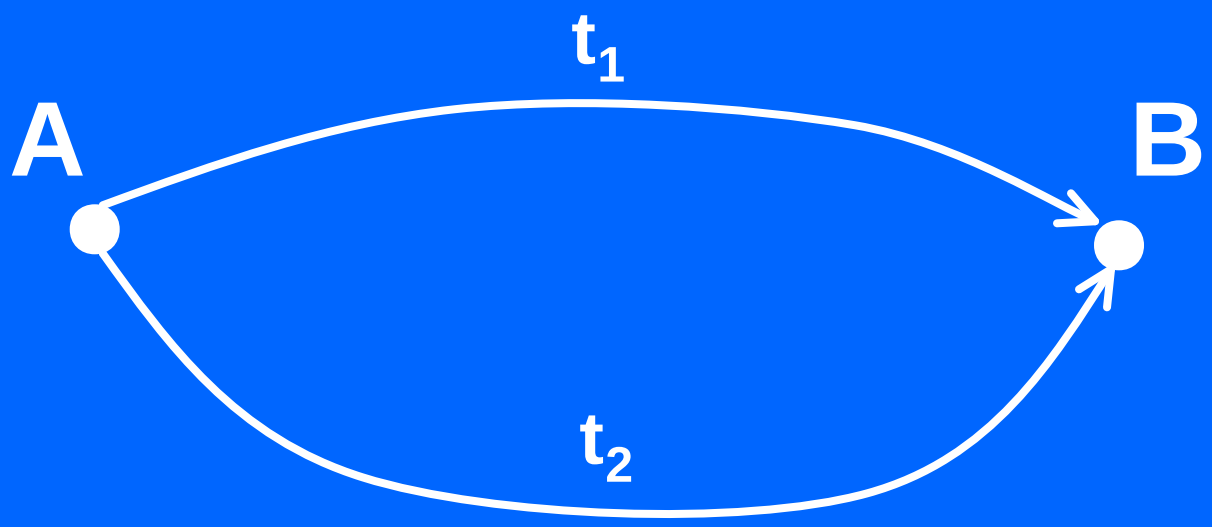


Assignment: A Brief History

- **Wardrop (1952):**

  “The costs on all routes used between any given pair of end points are equal and not greater than the cost experienced by a single vehicle on any unused route between them.”

Assignment: A Brief History



Initially: $t_1 < t_2$

At Equilibrium: $t_1 = t_2$

Assignment: A Brief History

Beckmann, McGuire & Winsten:

Studies in the Economics of Transportation (1956)

Assignment: A Brief History

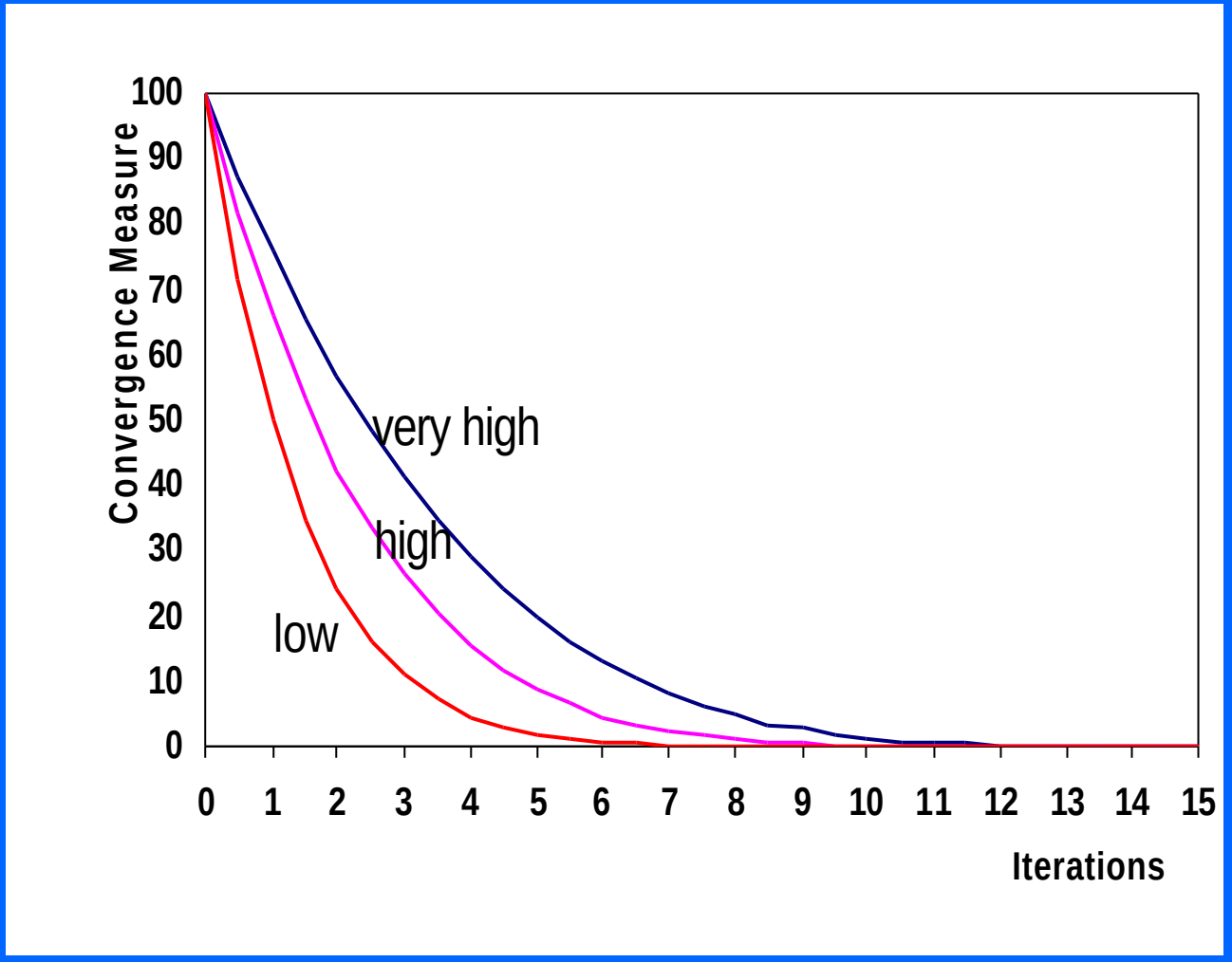
- **Approximate Methods:**

  - **Incremental Assignment**
  - **Capacity Restraint**

Equilibrium Assignment:

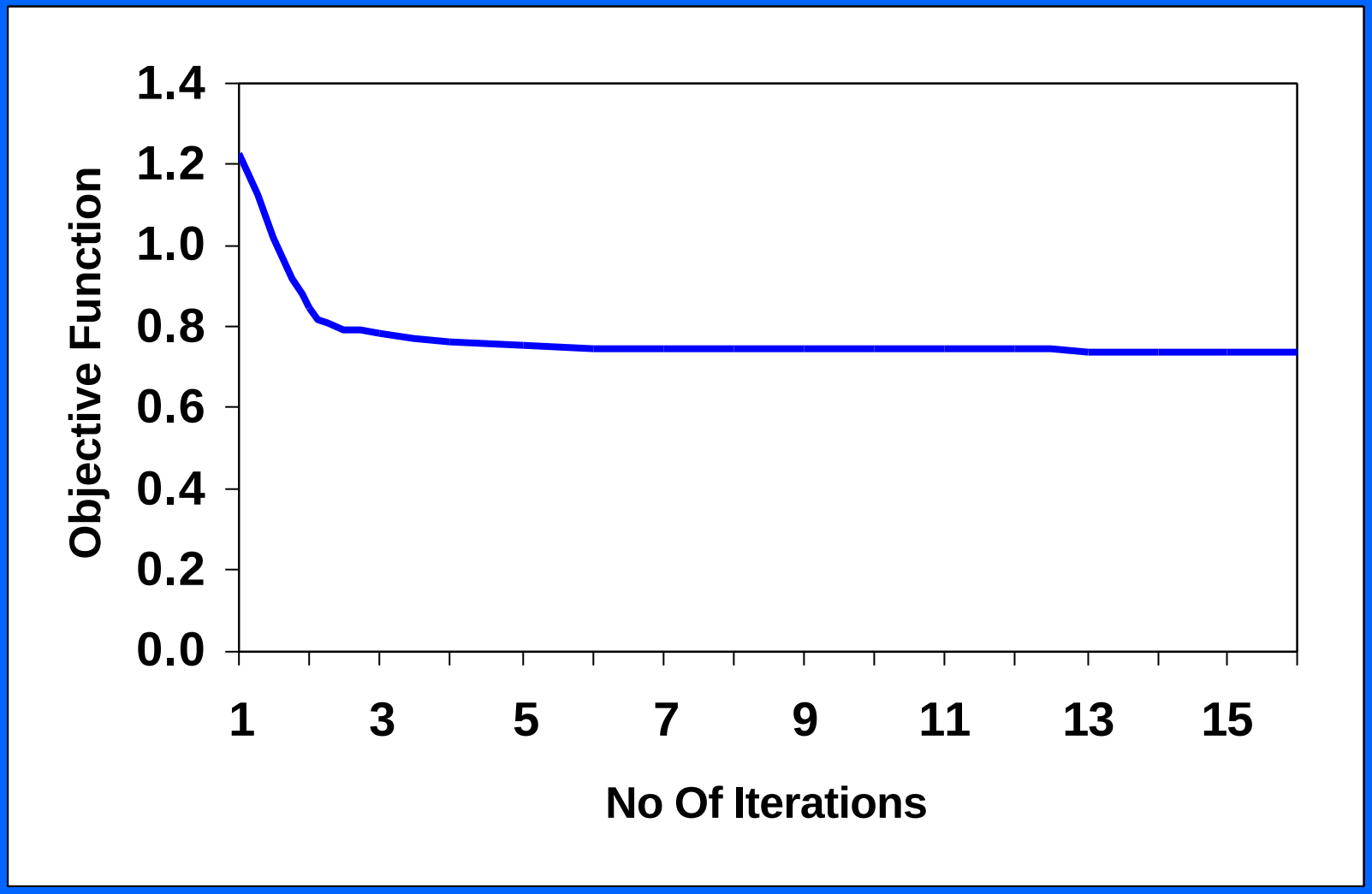
Done using the Franke-Wolfe algorithm

Consists of successive all-or-nothing assignments which are combined mathematically so as to minimize an objective function. The travel times are recalculated using the combined assignments and the process repeated.



From *Urban Transportation Networks* by Sheffi

From Gautrans Model



Stopping Criteria in EMME/2

- **Number of Iterations (15)**
- **Relative Gap (0.50)**

  An estimate of the difference between the current assignment and a perfect equilibrium assignment

- **Normalized Gap (0.50)**

  The difference between the mean trip time of the current assignment and the mean minimal trip time. The mean trip time is the average trip times used in the previous iteration while the mean minimal trip time is the average time computed using the shortest paths of the current iteration.

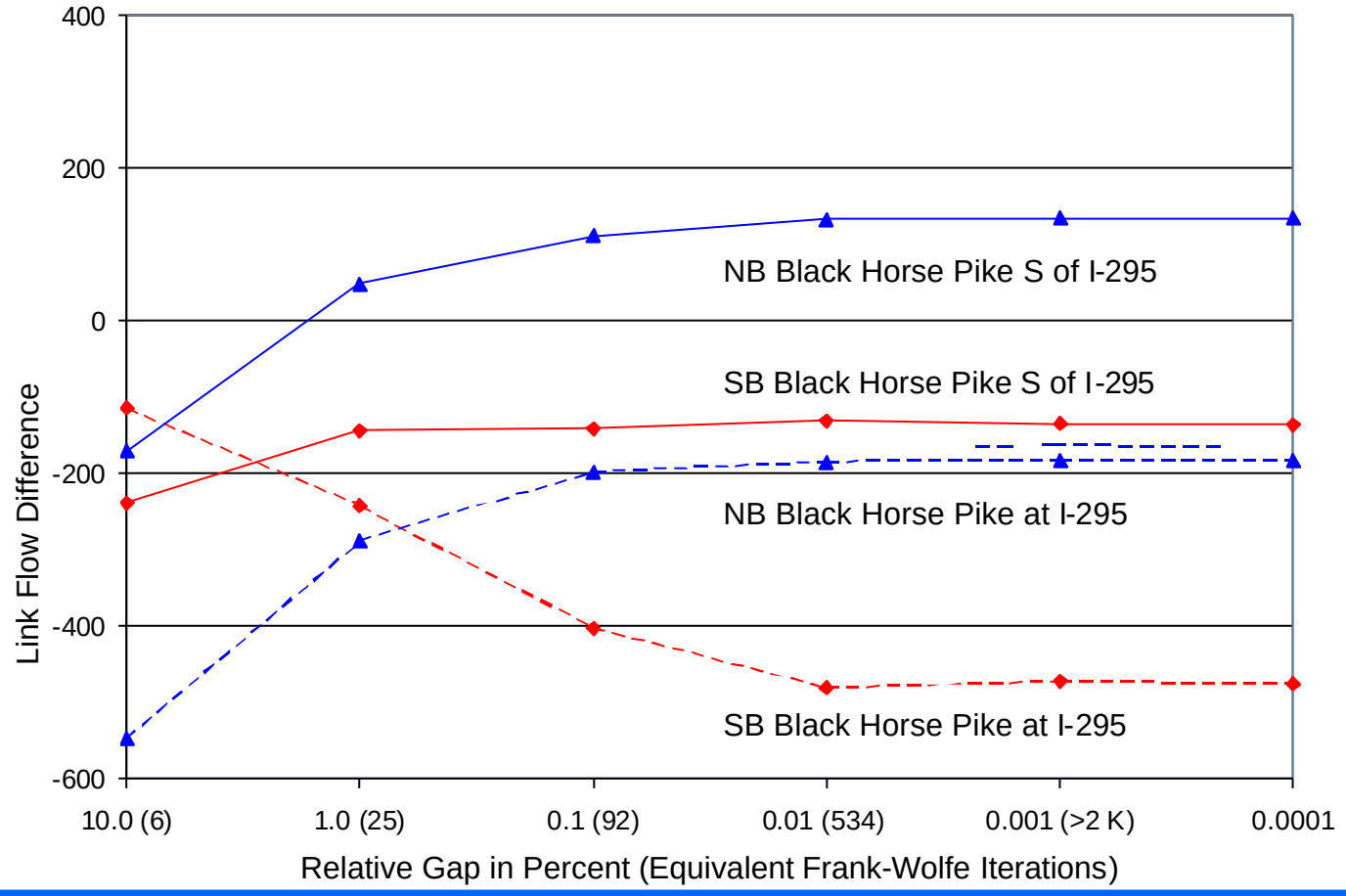
Convergence of Traffic Assignments: How Much Is Enough? The Delaware Valley Case Study

by

David Boyce, Biljana Ralevic-Dekic & Hillel Bar-Gera

EMME/2 User Group Meeting Albuquerque 2002

Fig. 6. Link Flow Differences (Build less No-Build) vs. Relative Gap
Black Horse Pike Crossing I-295 East of NJ 42



Reasons For Doing Assignments

- For economic evaluation
- For geometric and pavement design

Iterations	Ranking of Projects											
	1	2	3	4	5	6	7	8	9	10	11	12
15	12	7	25	10	28	17	23	21	11	1	31	3
16	12	25	10	7	28	17	11	23	21	1	31	14
17	12	25	10	7	28	17	23	11	21	1	31	14

Iterations	Ranking of Projects											
	1	2	3	4	5	6	7	8	9	10	11	12
15	12	7	25	10	28	17	23	21	11	1	31	3
16	12	25	10	7	28	17	11	23	21	1	31	14
17	12	25	10	7	28	17	23	11	21	1	31	14
23	28	5	12	10	11	17	22	25	21	1	23	2
24	28	5	25	30	12	10	17	22	32	7	23	21
25	5	30	25	17	10	32	7	12	28	22	23	21

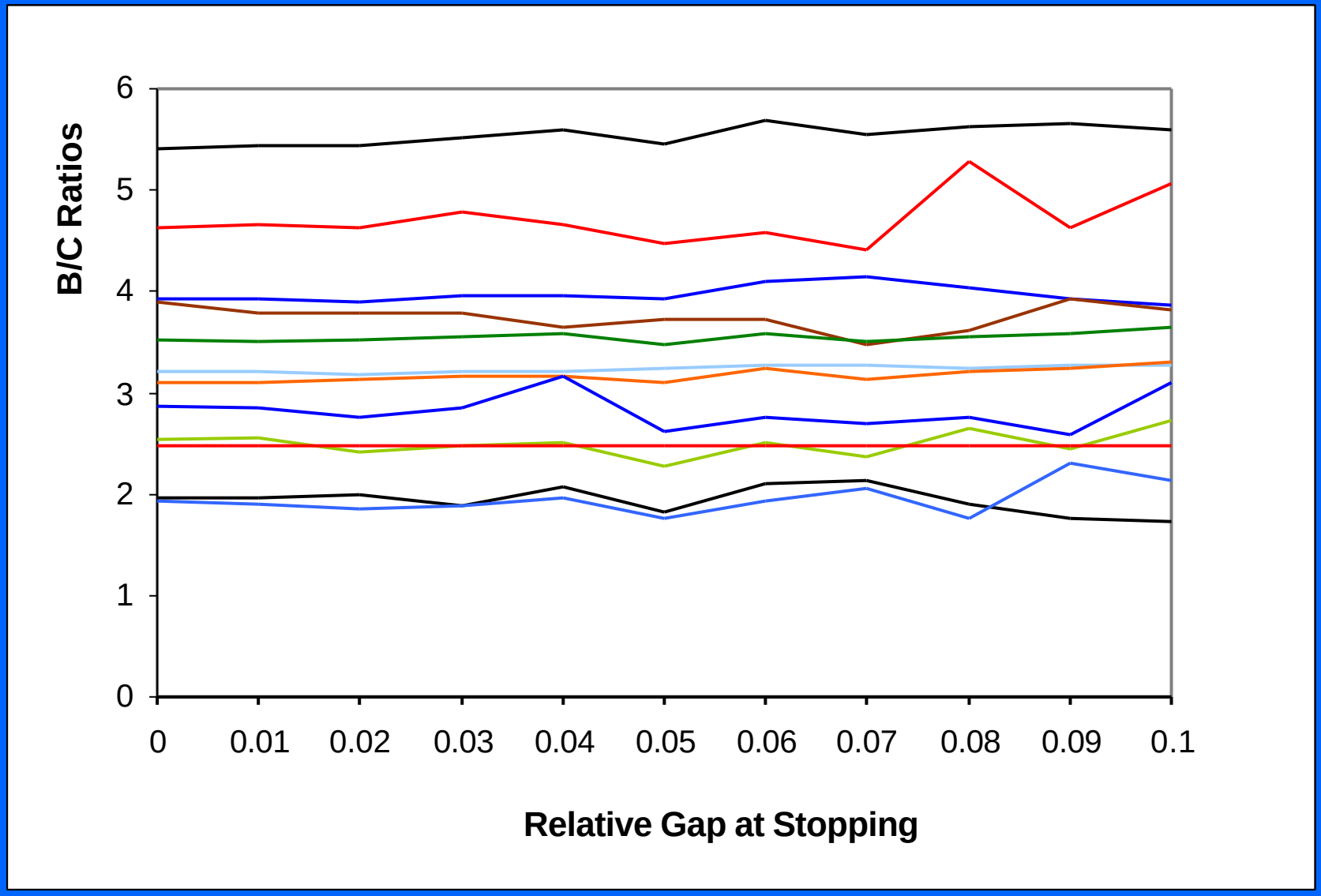
Iterations	Ranking of Projects											
	1	2	3	4	5	6	7	8	9	10	11	12
15	12	7	25	10	29	17	23	21	11	1	32	3
16	12	25	10	7	29	17	11	23	21	1	32	14
17	12	25	10	7	29	17	23	11	21	1	32	14
23	28	5	12	10	11	17	22	25	21	1	23	2
24	28	5	25	30	12	10	17	22	32	7	23	21
25	5	30	25	17	10	32	7	12	28	22	23	21
28	30	25	32	5	7	11	1	24	2	22	21	23
29	30	25	32	5	11	7	17	10	19	28	16	23
30	30	25	32	5	17	11	7	10	28	19	16	23

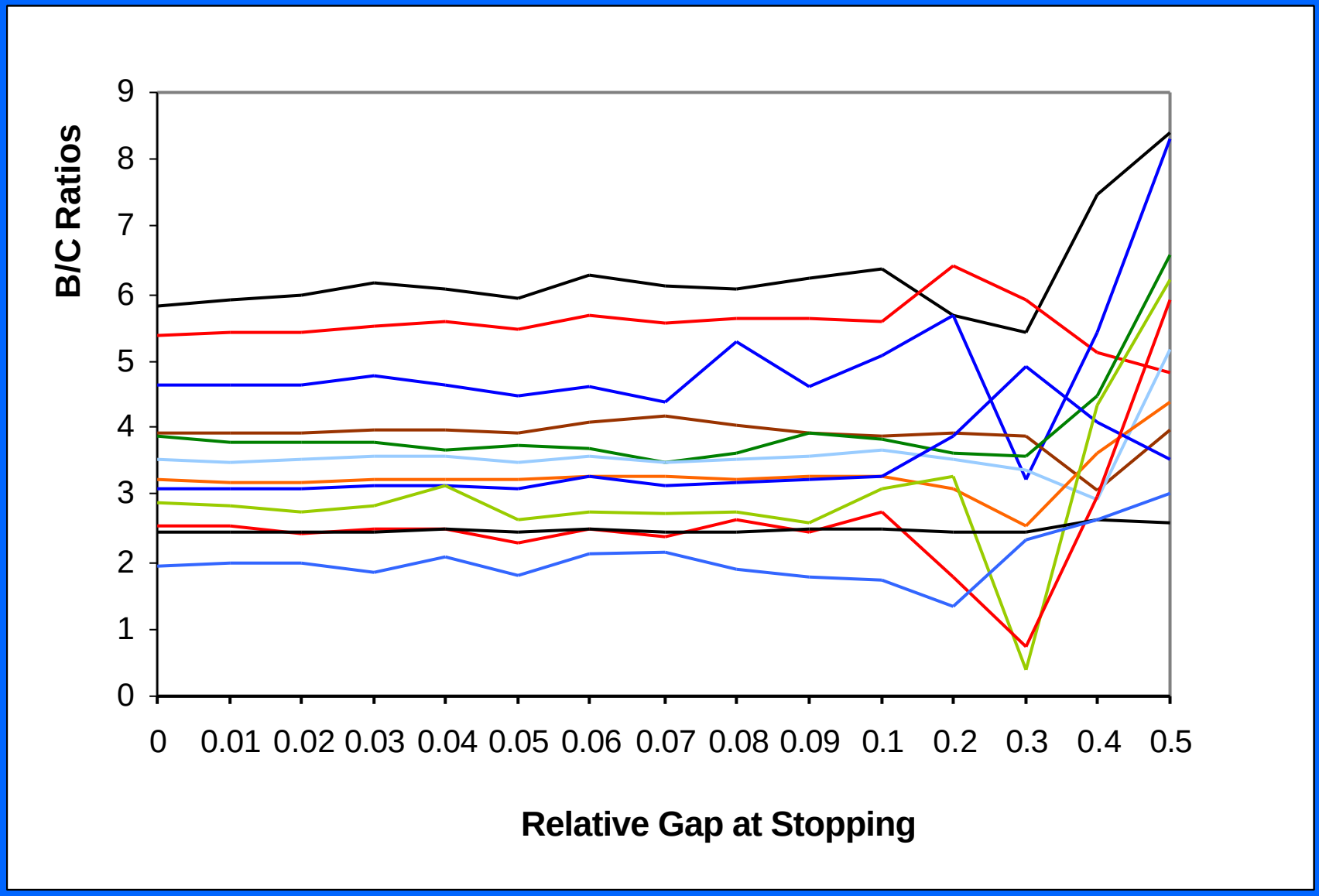
Relative Gap	Ranking of Projects											
	1	2	3	4	5	6	7	8	9	10	11	12
1.00	11	25	7	17	6	21	24	14	4	12	16	28
0.50	25	17	10	14	28	7	16	1	13	24	23	21
0.40	17	10	7	11	25	21	28	32	23	12	31	1

Relative Gap	Ranking of Projects											
	1	2	3	4	5	6	7	8	9	10	11	12
1.00	11	25	7	17	6	21	24	14	4	12	16	28
0.50	25	17	10	14	28	7	16	1	13	24	23	21
0.40	17	10	7	11	25	21	28	32	23	12	31	1
0.30	7	25	12	21	28	23	17	31	3	11	24	18
0.20	7	17	25	21	12	28	23	10	31	3	18	1
0.10	25	7	17	21	28	23	12	31	10	1	3	16

		Ranking of Projects											
		1	2	3	4	5	6	7	8	9	10	11	12
Relative Gap	1.00	11	25	7	17	6	21	24	14	4	12	16	28
	0.50	25	17	10	14	28	7	16	1	13	24	23	21
	0.40	17	10	7	11	25	21	28	32	23	12	31	1
	0.30	7	25	12	21	28	23	17	31	3	11	24	18
	0.20	7	17	25	21	12	28	23	10	31	3	18	1
	0.10	25	7	17	21	28	23	12	31	10	1	3	16
	0.05	25	7	17	21	28	23	31	12	10	3	1	11
	0.01	25	7	17	21	28	23	31	12	10	1	3	11

Relative Gap	Ranking of Projects											
	1	2	3	4	5	6	7	8	9	10	11	12
0.10	25	7	17	21	28	23	12	31	10	1	3	16
0.09	25	7	17	21	28	23	31	12	10	3	1	16
0.08	25	7	17	21	28	23	31	12	10	1	3	18
0.07	25	7	17	21	23	28	31	12	10	3	1	11
0.06	25	7	17	21	28	23	31	12	10	1	3	11
0.05	25	7	17	21	28	23	31	12	10	3	1	11
0.03	25	7	17	21	28	23	31	12	10	1	3	16
0.02	25	7	17	21	28	23	31	12	10	3	1	11
0.01	25	7	17	21	28	23	31	12	10	1	3	11
‘Ult’	25	7	17	21	28	23	31	12	10	1	3	11


B/C Ratios
6
5
4
3
2
1
0
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1
Relative Gap at Stopping


B/C Ratios
9
8
7
6
5
4
3
2
1
0
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1
0.2
0.3
0.4
0.5
Relative Gap at Stopping

Projects Not Ranked in the Top 12

Project No	Final Ranking	Final B/C Ratio	Highest Ranking	Stopping Criterion at Highest Ranking
2	21	0.46	5	RGap = 1.40
4	20	0.59	5	RGap = 1.80
5	31	-0.32	1	25 Iterations
6	16	1.37	5	RGap = 1.20
14	24	0.37	4	RGap = 1.55
19	32	-1.26	1	RGap = 1.60
24	14	1.77	2	RGap = 1.40
30	29	-0.01	1	29 Iterations
32	25	0.36	3	17 to 30 iterations

Regression Coefficients

	Iterations		Relative Gap			
	15	30	1.0	0.5	0.1	0.01
R^2	0.734	0.730	0.728	0.728	0.727	0.727
Intercept	21.24	18.52	20.96	18.48	17.39	17.96
Slope	0.780	0.776	0.775	0.774	0.773	0.773

Cumulative Percentage Within Given Percentage of “Equilibrium” Link Volumes

	Iterations		Relative Gap						
	15	30							
0	18.1	20.1							
0.1	18.5	21.3							
1	31.1	40.0							
2	43.1	54.9							
5	64.2	75.6							
10	79.2	87.6							
25	92.0	95.6							
50	96.4	97.8							
100	98.4	99.0							

Cumulative Percentage Within Given Percentage of “Equilibrium” Link Volumes

	Iterations		Relative Gap						
	15	30	1.0	0.5	0.4	0.3	0.2	0.1	0.01
0	16.5	21.7	19.2	23.4	25.0	27.4	30.8	37.7	69.6
1	33.7	47.8	45.0	55.5	59.0	64.0	70.5	80.5	97.3
2	46.7	63.5	61.5	71.9	74.9	79.1	83.7	90.2	98.9
5	69.7	83	82.1	88.4	90.0	92.0	94.1	96.7	99.7
10	83.9	91.9	91.8	94.9	95.7	96.7	97.6	98.7	99.9
25	94.4	97.5	97.5	98.5	98.7	99.1	99.4	99.7	100
50	97.7	98.9	99.0	99.4	99.6	99.7	99.8	99.9	100
100	99.0	99.5	99.6	99.8	99.8	99.9	99.9	100	100

Computational Effort

Relative Gap	No. of Iterations	Time(m:s)
0.01	692	40:10
0.02	376	22:12
0.03	275	16:28
0.04	218	13:26
0.05	184	10:58
0.10	113	6:44
0.20	72	4:16
0.50	40	2:24

Conclusions

- Be wary of using default values for stopping criteria
- An objective measure of convergence is preferable to number of iterations
- Different stopping criteria can be used depending on purpose of assignment

Recommendations

- **For Economic Evaluation:**
  R Gap = 0.01
- **For Link Volumes:**
  Use Table 4 to provide a "level of confidence"
- **Do not use number of iterations**

Final Remarks

- Thanks to Gautrans
- My own views